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POLYPHASE CURRENTS.

BY

ALFRED STILL,

ASSOC.M.INST C.E., M.I.E.E.,

AUTHOR OF "ALTERNATING CURRENTS AND THE THEORY OF TRANSFORMERS."

WITH NUMEROUS DIAGRAMS.

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PREFACE.

With the extended use of polyphase alternating currents for the transmission and distribution of electric power, there would seem to be a demand for a book treating of the theoretical considerations involved in polyphase working in such a manner as to commend itself to practical engineers and those students who are without the mathematical knowledge required for the study of the more advanced works on the subject.

The author has adopted a non-mathematical treatment of the subject throughout, but has made extensive use of graphical methods. The introductory chapters are written with the object of explaining the use of vectors in solving alternating-current problems, and also for the purpose of drawing attention to the chief points of difference between alternating and continuous currents of electricity. It should, therefore, be possible for any student or engineer with a fair knowledge of continuous-current working, but with only a superficial acquaintance with alternating currents, to obtain a thorough grounding in the principles underlying polyphase working, provided he will take the trouble to master the contents of the first two chapters.

The few notes added at the end of the book, in the form of an appendix, deal with certain matters referred to in the text, but which are not essential to the scheme of the book, or to the elucidation of subsequent chapters.

It may be urged that the subject of electric power transmission has been given undue prominence; but the reason for the somewhat disproportionate length of this section is that the book has not been primarily written for the use of *designers* of polyphase machinery (for these must have a very thorough and detailed knowledge of the subject), but rather for the *user*, or the engineer who has to lay out and to work complete polyphase schemes. If exact details are required of any particular piece of polyphase machinery, this information can always be obtained from the makers of the plant.

Again, Kelvin's law and the principles determining the most economical section of conductors are discussed at some length, not only because they have a very important bearing on the subject of polyphase transmission of power, but also because they do not appear to be generally understood. These questions have been treated in a non-mathematical manner; but the main points at issue have been clearly stated, and the reader should have no difficulty in calculating the most economical size of conductor to suit any given electric transmission scheme.

The author desires to thank the editors of the *Electrical Engineer* and of the *Electrical Review* for the loan of blocks and permission to reprint portions of certain articles that have appeared in these journals.

ELLESMERE PARK,
ECCLES, LANCASHIRE

June, 1906.

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POLYPHASE CURRENTS.

CHAPTER I.

ELEMENTARY STUDY OF ALTERNATING CURRENTS.

IN order to lead up to the subject of polyphase alternating currents, it will be advisable to consider, in the first place, the general principles which underlie the working of all alternating currents of electricity, whether single phase or polyphase; and it is proposed to devote this and the following chapter to an elementary study of single-phase currents; the object being to point out the essential differences between direct and alternating currents, and explain the various effects peculiar to alternate-current working.

(1) **Definition.** — An alternating current is — as indicated by the name—a current which, instead of being unidirectional, such as the current produced by a battery or a continuous-current dynamo, flows first in one direction and then in the opposite direction in regular succession. In other words, it is a current which, starting from zero value, increases in strength in a *positive* direction until it reaches its maximum positive value; it then dies down to zero value, after which it rises to its maximum *negative* value, and again falls to zero; this process being repeated in a periodic manner.

(2) **Generation of Alternating Currents.**—There is little difference in principle between an alternator

and a direct-current dynamo; it is the absence of the commutator in the alternating-current generator which leads to the current obtained from the terminals or collecting rings being of the character described above. Given any continuous-current generator, it is evidently merely necessary to replace the commutator by a couple of slip rings connected to opposite points on the armature winding, and to excite the field coils from a separate continuous-current source, in order that currents alternating in direction may be drawn from the armature.

There are many types of alternator which differ little in appearance from multipolar direct-current dynamos, and it will readily be understood that the number of reversals of the current in a given time will depend upon the speed of rotation of the armature and the number of poles in the exciting field.

(3) **Graphical Representation of an Alternating Current.**—Let Fig. 1 represent the chart of a centre zero recording ammeter, such as might be used for registering the charge and discharge currents of a battery. On such a chart the lapse of time is measured horizontally, from left to right, while the strength of the current is indicated by vertical distances above or below the horizontal centre line. If the distance is measured *above* this horizontal datum line, this is an indication that the current is flowing in a *positive* direction, corresponding, let us say, with a charging current; but if the measurement is made *below* the datum line, this indicates that the current is flowing in a *negative* direction, and that the battery is being discharged.

Referring to the curve in Fig. 1, it will be noticed that, between the times indicated by the points t and t_1 on the datum line, corresponding to a period of 4 hours and 30 minutes, the charging current has gradually fallen from 50 amperes down to 25 amperes, after which it has increased in strength, only to be entirely interrupted shortly afterwards before gradually increasing again, but this time in a *negative* direction—*i.e.*, as a discharging current.

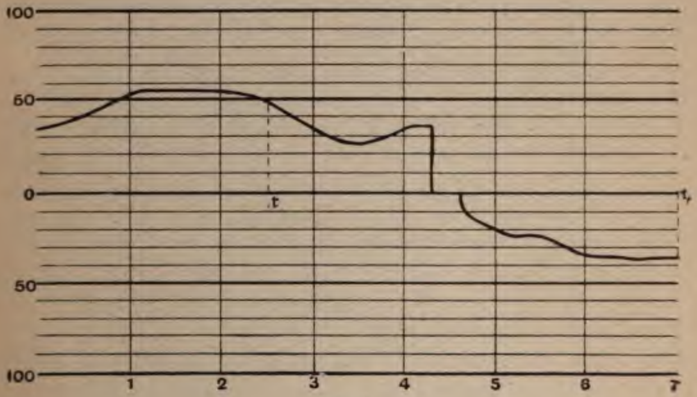

FIG. 1.

The variation of an alternating current, both in strength and direction, is commonly represented in a similar manner. In such a curve as that shown in Fig. 2, the lapse of time is measured—as in the previous example—horizontally from left to right, while the strength and direction of the current, at any particular instant, are indicated by the length or the vertical ordinate and its position relatively to the datum line. The chief difference between this and the previous diagram (Fig. 1) lies in the fact that, whereas in the first diagram we were dealing with a lapse of time measured by hours, the total horizontal distance in Fig. 2 covers only a small fraction of a second of time; and, further, the nature of the curve, instead of being irregular as in Fig. 1, is now such that each succeeding positive half-wave is exactly similar to the preceding one, and each succeeding negative half-wave is also exactly similar to the one that came before. In fact, the current we are now considering is a *periodic function* of the time: the current rises and falls through a certain cycle of values which is repeated over and over again; and the *periodic time*, or time required for the performance of one complete cycle, is represented in Fig. 2 by the horizontal distance O *b*. This distance, as already mentioned, represents—in the case of the

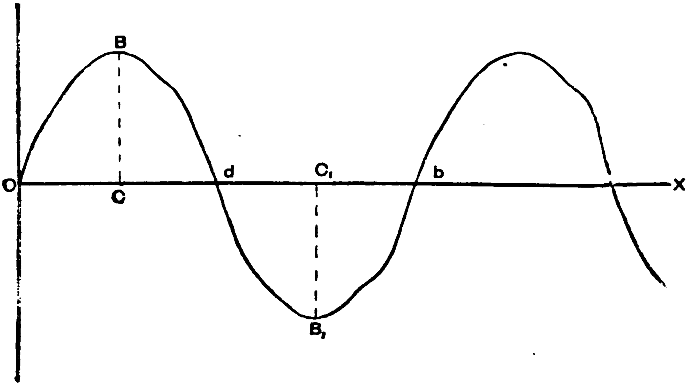

FIG. 2.

currents we are at present concerned with—only a small fraction of a second of time; and what is known as the *periodicity* of an alternating current is the number of complete cycles performed in one second.

(4) **Frequency.**—The word *frequency*, when used in connection with alternating currents, is synonymous with periodicity, and either word denotes the number of periods or complete cycles per second.

Thus, in Fig. 2, if the distance O b is equivalent to $\frac{1}{n}$th of a second, the periodicity or frequency of the current represented in the diagram will be n, and the number of *reversals* or *alternations* per second will be $2\,n$; from which it follows that the use of the word *alternation* to denote a complete cycle is inaccurate, and it is always safer to speak of the *periodicity* of an alternating quantity, as this term admits of no ambiguity.

In practice the number of periods per second lies usually between 25 and 100. In the early days of alternating currents, when these were used solely for lighting purposes, the periodicity was commonly about 100 in this country; but there are reasons which render a lower frequency advisable, and from 80 to 50, or even lower, is the usual practice nowadays.

With the introduction of motors, both single-phase and polyphase,* there were additional reasons for still further lowering the periodicity; and, indeed, if it were not that motors are frequently driven off the lighting circuits, they would probably be wound for still lower periodicities, the present limit—except in the case of large power transmission schemes—being the point at which the alternations of the current become noticeable to the eye.

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* The meaning of these expressions, if not familiar to the reader, will be made clear to him in future pages.

(5) **Mean and $\sqrt{\text{Mean Square}}$ Values of Alternating Currents.**—Referring again to Fig. 2 (which for convenience is reproduced on this page), it will be readily understood that this curve may

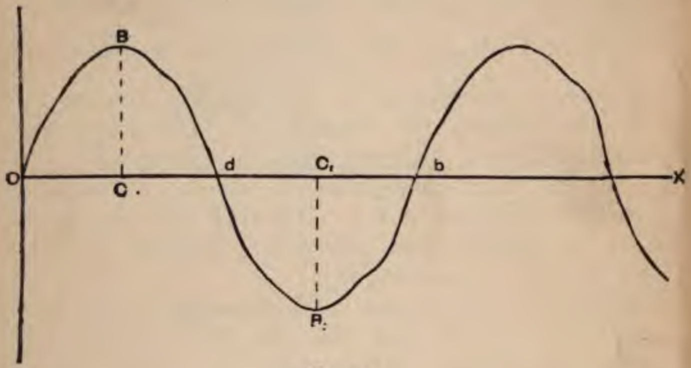

FIG. 2.

stand for any alternating quantity, such as the E.M.F. of a generator, or the potential difference between the terminals of an incandescent lamp; and, in any case, whether the curve represents the periodic variation of amperes or volts, the length of the ordinate C B is a measure of the *maximum* positive value of the alternating quantity, and the distance C_1 B_1 is a measure of the maximum negative value.

In practice the negative half-wave is generally similar in shape and equal in magnitude to the positive half-wave, and it therefore follows that the ordinates C B and C_1 B_1 are equal in length. With this *maximum* value of an alternating quantity we shall not concern ourselves at present; it will suffice to point out that this value of an alternating *current* will determine the total magnetic flux, or number of magnetic lines linked through the circuit at each reversal; and the insulation of the circuit must be considered in relation to the maximum value of the *voltage* curve.

With respect to the *mean* value of an alternating current, it might be supposed that this is the quantity with which we shall be principally concerned; but, as a matter of fact, this is not the case. In the design of alternators, transformers, electromagnetic measuring instruments, etc., it is undoubtedly this value of the induced E.M.F. which is most easily calculated, but without a knowledge of the *shape* of the wave, as shown in Fig. 2, we shall still be without some most important information. It should, perhaps, be hardly necessary to explain what is meant by the *mean* value of an alternating current or E.M.F.: we have merely to take the average of all the ordinates of the wave diagram, or—what amounts to the same thing—measure the area of the curve O B *d* (Fig. 2) and divide by the length O *d*. If a planimeter is not available, recourse can be had to the method frequently adopted in connection with steam-engine diagrams, that is to say, the half-period O *d* would be divided into a convenient number of equal parts, and the sum of the lengths of all ordinates erected on the central point of every section, when divided by the total number of such ordinates, will approximate to the true *mean* value of the current or E.M.F., as the case may be.

Referring still to Fig. 2—which we shall assume represents the variation in strength and direction of an alternating *current* of electricity—let us suppose that the resistance of the circuit conveying the current is R ohms; now, if C is the value of the current at any instant, the rate at which work is being done in *heating* the conductors will, at that particular instant, be equal to C^2 R. Hence, if we wish to know the *average* rate at which work is being done by an alternating current, we must calculate the mean value of the *square* of the current and multiply this quantity by the resistance R. Hence, instead of taking the mean of a large number of ordinates of the half-wave O B *d*—as in the previous example — we must now take the average of the *squares* of all such ordinates, and this quantity, multiplied by the ohmic resistance of the circuit, will give us the watts lost in heating the conductors.

It follows, therefore, that when we speak of an alternating current as being equal to a certain number of amperes, we invariably allude to that value of the current which, when squared and multiplied by the resistance of the circuit in which it is flowing, will give us the actual power in watts which is being spent in overcoming the resistance of the conductors.

Thus it is the *root-of-the-mean-square* value of an alternating current which, as far as *power* measurements are concerned, enables us directly to compare a periodically varying current with a continuous current of constant strength: it is the product of this value of the current and the corresponding value of the effective or resultant E.M.F. to which it owes its existence which, in all cases, is a measure of the power absorbed in the circuit.

The readings of nearly all commercial measuring instruments for alternating currents depend upon the $\sqrt{\text{mean square}}$ value of the current or E.M.F.; and it is only in exceptional cases that we require to know either the *maximum* or the true *mean* value of an alternating quantity.

If we apply a potential difference of 100 volts to the terminals of an incandescent lamp—the inductance of which is very small, and practically negligible*—the lamp will glow with the same brilliance whether this potential difference is obtained from a continuous or an alternating current source; and when we speak of an alternating E.M.F. of 1 volt, it is the $\sqrt{\text{mean square}}$ value of the alternating voltage to which we refer. Again, when we speak of an alternating current of, say, 10 amperes, we invariably refer (unless special mention is made to the contrary) to the $\sqrt{\text{mean square}}$ value of the periodic current which, so far as heating effects are concerned, is exactly equivalent to a continuous current of 10 amperes.

(6) **Phase Difference.**—Two alternating E.M.F.'s of the same character and periodicity, or an alternating current and the E.M.F. to which it owes its existence, are said to be *in phase* when the growth

---

* The great importance of the amount of the induction in a circuit conveying an alternating current will be fully dealt with in the following chapter.

and decrease, reversal, and maximum values (of the same sign) occur simultaneously.

Thus, in Fig. 3, let the full line curve C represent the periodic variations of the current flowing in a given circuit, and let the dotted curve E represent the alternating E.M.F. in the circuit: the periodicity of these two quantities is evidently the same, as indicated by the length of the line *o t*—representing the time of one complete period—being equal in both cases. Moreover, it will be observed that the current and E.M.F. pass through zero value, reach their maximum positive value, reverse their direction, and pass through their maximum negative value at

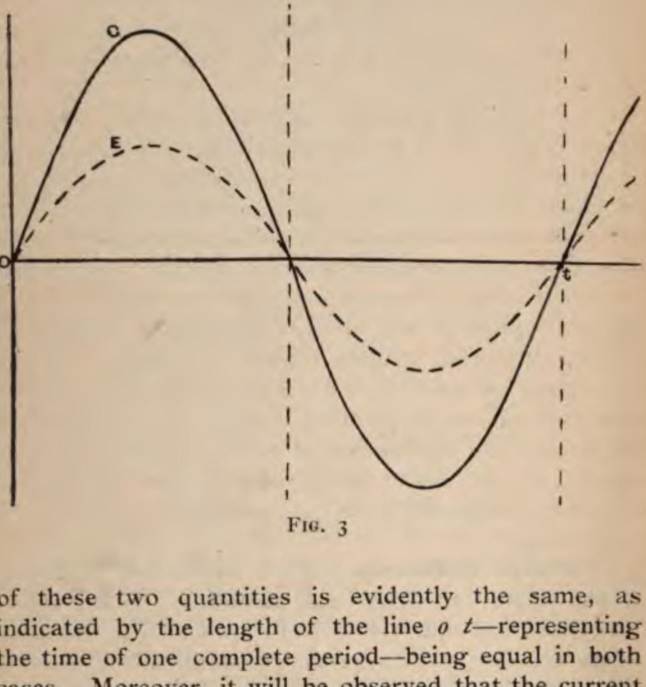

FIG. 3

precisely the same instant of time, and they are therefore *in phase.*

Consider, now, the two curves drawn in Fig. 4.

We shall assume that these represent two E.M.F.'s which, for the sake of the argument, might be produced by two alternators, similar in all respects, and having their shafts rigidly coupled together.

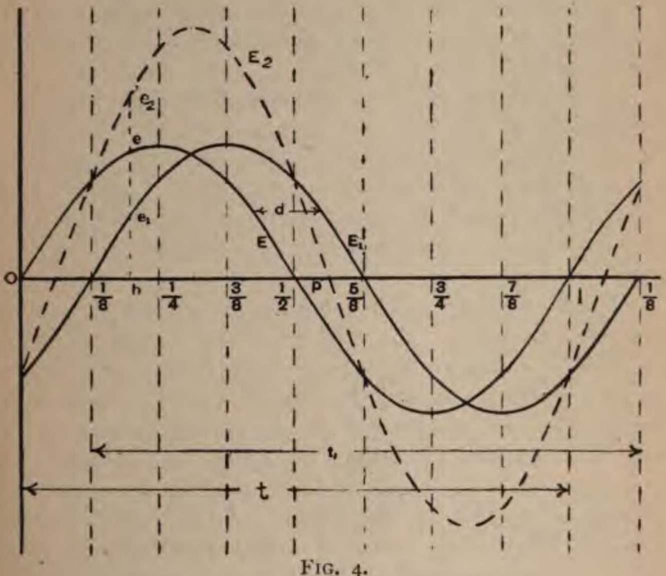

FIG. 4.

These two E.M.F.'s are of the same periodicity—the distances t and t_1 being equal — but in this case the two alternating quantities are no longer *in phase.* It will be noted that, throughout the complete cycle, the instantaneous values of the

E.M.F. represented by the curve E_1 occur exactly one-eighth of a period after the corresponding values of the curve E; and E_1 is therefore not in phase with E, but *lags behind* this E.M.F. by a small fraction of a second represented by the distance *d*, which, in this example, is exactly equal to one-eighth of a complete period. In other words, there is a *difference of phase* between these two E.M.F.'s equal to one-eighth of a period.

(7) **Addition of Alternating E.M.F.'s.**—Let us suppose that the two alternators, with shafts rigidly coupled together, generating two alternating E.M.F.'s with a phase difference of one-eighth of a period, are electrically joined in series on the same circuit; and let us consider what will be the resultant E.M.F. in the circuit.

When dealing with continuous currents, it is merely necessary to add together the various E.M.F.'s—such as those due to a dynamo in series with a battery—paying due regard to their respective *signs*—*i.e.*, to whether they are acting in a positive or a negative direction—in order to obtain the resultant E.M.F. producing, or tending to produce, a flow of current in the circuit. And so also in the case of alternating currents *at any particular instant of time*, the resultant E.M.F. in any circuit is equal to the algebraical sum of the various E.M.F.'s in the circuit. Thus, on referring again to Fig. 4, it will be noticed that a dotted curve E_2 has been drawn in addition to the two curves E and E_1. Every ordinate of this curve, such as $h\,e_2$, is equal to the sum of the ordinates, $h\,e$ and $h\,e_1$, of the two full line curves; due attention being paid to the *sign* of these instantaneous values of the E.M.F.

For instance, when the positive value of E_1 is exactly equal to the negative value of E, the resultant E.M.F. in the circuit will be *nil*, and this is what occurs at the instant *p* where the resultant curve, E_2, passes through its zero value.

A cursory examination of this diagram (Fig. 4) will make it clear that the resultant E.M.F., E_2, is of the same periodicity as the two component E.M.F.'s; and not only its maximum value, but also its $\sqrt{\text{mean square}}$ value will be something less than would be obtained by merely adding together the corresponding values of the E.M.F.'s E and E_1.*

(8) **Vector Diagrams.**—The method described above for graphically adding together two (or more) periodically alternating quantities is a very tedious and altogether unpractical one. The method about to be described—and which involves the proper understanding of a *vector* quantity—is of such general utility in the solution of alternating-current problems that the reader who may not be familiar with these diagrams should devote his most careful attention to the following explanations.

In the first place, a *vector* quantity (as distinguished from a *scalar* quantity) possesses not only *magnitude*, but also *direction*.

A *magnitude* can be graphically represented by the *length* of a straight line, and a *direction* can be represented by the *angle* which a line (of indefinite length) makes with a datum line drawn on the same plan for the purpose of reference; but a vector quantity can only be graphically represented by a line of definite length drawn in a definite direction relatively to some datum line.

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* It is only when the two component waves are *in phase* and similar in shape, that the resultant E.M.F. will be exactly equal—both in regard to its maximum and its $\sqrt{\text{mean square}}$ values—to the simple addition of the corresponding values of the component waves.

The resultant magnetic force at any particular point in space is a vector quantity, because this force has not only a definite direction relatively to the earth's axis, but it also has a definite intensity; and if it were customary to indicate, on geographical maps, the magnetic condition at any given place, a line would have to be drawn not only in a certain direction, but also of a definite length such as to indicate, let us say, the intensity of the horizontal component of the earth's magnetism at that place. Such a line, drawn to scale, with an arrow head at one end (to indicate direction) would be what is commonly known as a *vector*.

Instead of representing an alternating current or E.M.F. by means of the wave diagrams with which we have been studying them up to the present, let us assume that we are in no wise concerned with the maximum or mean values, or the law of variation of an alternating quantity, but only with its $\sqrt{\text{mean square}}$ value, which, as explained above, is what we most frequently wish to know. This is also the quantity which is measured by an alternating-current ammeter or voltmeter. A straight line of such a length as to indicate the amount or intensity of this alternating quantity drawn *in any direction* on a sheet of paper would be a correct graphical representation of a definite number of amperes or volts. But when dealing with two or more alternating forces all acting in the same circuit, it is not only necessary that the straight lines representing these forces should be of definite lengths; they must also be drawn at certain definite angles relatively to each other in order to represent the phase differences between them.

In Fig. 5 the complete circle of 360 deg. (or 2π radians) represents one period, and in order to indicate the phase difference between the alternating quantities E and C, two straight lines O E and O C—starting from the common point O, and of such lengths as to indicate the respective *magnitudes* of these quantities—are drawn with an angle θ between them such that the ratio which this angle bears to the complete circle expresses the *phase difference* between E and C as a fraction of the complete period.

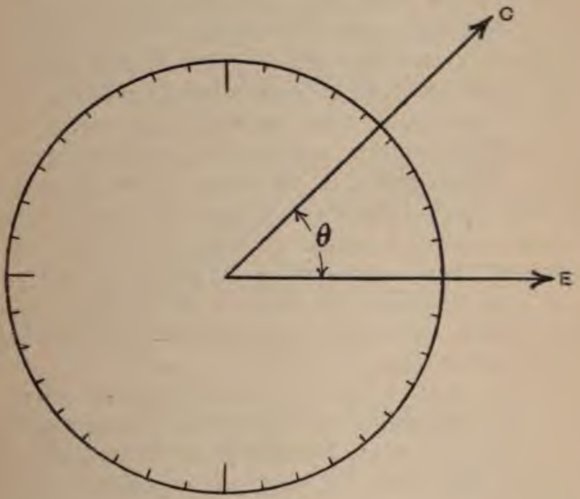

FIG. 5

For instance, if the angle θ (as in Fig. 5) is equal to 45 deg., or $\frac{\pi}{4}$ radians, this means that the current C *lags behind* the E.M.F. E by a time interval equal to one-eighth of a period; or, in other words, the E.M.F. E is *in advance* of the current C by this same interval of time.

Thus, if the *periodicity* of this particular E.M.F. is 40, it follows that the angle C O E in Fig. 5 represents a time interval of $\frac{1}{320}$th of a second.

It is not usual to express a phase difference as a definite interval of time, because this would convey no useful information, unless the periodicity were also stated. What we wish to know is the fraction of a complete period by which one of the alternating quantities is in advance of the other, and this, as explained above, can readily be stated as an angle; it is only necessary to bear in mind that the complete circle (360 deg.) stands for one period or double alternation.

If the reader has carefully followed what has been said regarding the graphic representation of alternating quantities by means of *vectors*, it is more than probable that he may yet experience considerable difficulty in forming a clear conception of the phase difference—not between the respective *maximum* or *zero* values, or any other definite instantaneous value of two alternating quantities, but between their *average* or, to be more specific, their $\sqrt{\textit{mean square}}$ values.

This difficulty is dealt with in the following article.

(9) **Addition of Alternating E.M.F.'s with the Use of Vector Diagrams.**—Consider once

again (as in articles 6 and 7) two alternators, A and B, joined in series: and assume that they have the same number of poles, and are driven at the same speed.

Let us suppose three voltmeters to be connected as shown in Fig. 6. These voltmeters must be such as may be used indifferently on alternating or direct current circuits; that is to say, they must measure the $\sqrt{\text{mean square}}$ values of the alternating volts. They may, for instance, be either electrostatic or hot-wire instruments; but they should not depend upon the electromagnetic actions of coils having iron cores, because the induction in the iron cores will depend upon the *mean* value of the applied E.M.F., and the readings of such instruments will, therefore, be more or less dependent upon the wave form of the impressed volts.

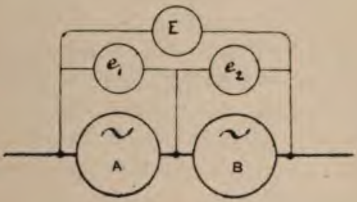

FIG. 6.

The voltmeters e_1 and e_2 will give us the volts due respectively to the alternators A, and B, whereas E will measure the resultant volts at terminals. If the volts measured by E are equal to the arithmetic sum of the volts e_1 and e_2, the two machines would be said to be *in phase*, although it is quite possible that the *maximum* values of the component E.M.F.'s might not be reached at the same instant of time, if the waves produced by the two alternators are not similar in form. As a rule, the reading on E will be *less* than the arithmetic sum of e_1 and e_2. We will suppose these three values to be known. From the centre O (Fig. 7) describe a circle of radius O E, the length of which is a measure of the volts E. Now draw $o\ e_1$ in any direction to represent the volts e_1. From e_1 as a centre describe an arc of radius e_1 E, the length of which is proportional to the volts e_2: it will cut the arc already drawn at the point E. Join O E, and complete the parallelogram O e_1 E e_2. The angle θ between the two component vectors $\overline{o\ e_1}$ and $\overline{o\ e_2}$ is a measure of what we must now understand as the

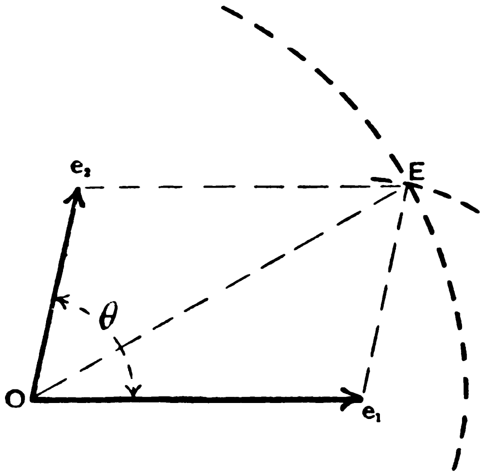

FIG 7.

angle of lag or *phase difference* between two alternating quantities which are of the same frequency, but which do not necessarily follow the same law of variation.

Bearing in mind the above definition of the phase difference or angle of lag between two alternating quantities—let us say two E.M.F.'s—of the same periodicity, in a given circuit, it will be readily understood that, in the case of the two E.M.F.'s being *in phase*, the vectors $\overline{o\,e_1}$ and $\overline{o\,e_2}$ will lie on the same straight line, and the total or applied E.M.F., E, will be exactly equal to the sum of the two component E.M.F.'s, e_1 and e_2.

Again, if e_1 and e_2 are exactly *opposite* in phase, the vectors $\overline{o\,e_1}$ and $\overline{o\,e_2}$ will still lie on the same straight line, but instead of acting in the same direction, they will now act against each other, and the resultant $\overline{\mathrm{O\,E}}$ will be equal to the *difference* between these two forces, while the angle of lag is equal to half a circle, or 180 degrees.

An inspection of the wave diagram (Fig. 4, p. 19) will make it clear that this is what must actually occur in the case of the two particular values of the phase difference which have been chosen in order to help the reader towards a clear understanding of the diagram Fig. 7.

When referring to Fig. 5 (p. 23), the E.M.F., E, was spoken of as being in *advance* of the current C; and, again, the vector $\overline{\mathrm{O\,C}}$ was said to represent a current *lagging behind* the E.M.F., E. It is merely a matter of convenience to so represent the leading and lagging alternating quantities. The distinction is easily remembered on account of the similarity of such a diagram to a clock face, the hands of which

3*

are set *forward* by moving them from C to E and *back* by moving them from E to C.

If the diagram Fig. 7 has been correctly drawn, it is, therefore, the E.M.F. e_1 which is in advance of e_2, and the total E.M.F., E, is in advance of e_2, but lags behind e_1.

The question of compounding two or more alternating forces in an electric circuit now becomes a very simple matter. Thus, in Fig. 7, had we been given the two voltages e_1 and e_2 and the phase difference θ (instead of the three voltages), we could have calculated the total E.M.F., E, and ascertained its phase relation to the two component forces, by merely constructing the *triangle of forces* $o\ e_1$ E in the manner familiar to every engineer.

As an example, let us suppose that there are three distinct alternating E.M.F.'s—A, B, and C—of the following values, all combining to produce one resultant E.M.F. in an electric circuit:

A = 200 volts.
B = 150 volts.
C = 100 volts.

We shall also assume that B lags behind A by exactly a quarter of a period (90 deg.), while C leads, or is in advance of A, by the fraction of a period denoted by an angle of 35 degrees.

Draw the three vectors $\overline{\text{O A}}$, $\overline{\text{O B}}$, and $\overline{\text{O C}}$, in Fig. 8, to a suitable scale, and such that the angles A O B and A O C are respectively equal to 90 deg. and 35 deg., bearing in mind that O B must be drawn *behind* O A, while O C must be drawn *in advance*. Construct the *polygon of forces* O B r R closed by the *resultant* $\overline{\text{O R}}$, which represents the sum of the three forces—or vectors—$\overline{\mathrm{O\,A}}$, $\overline{\mathrm{O\,B}}$, $\overline{\mathrm{O\,C}}$, and which *lags behind* the vector $\overline{\mathrm{O\,A}}$ by an angle θ equal, in this example, to 18 deg. The measured length of the line O R to the same scale as the three component vectors gives us the value (in volts) of the resultant E.M.F. In this example it will be found to be 297 volts.

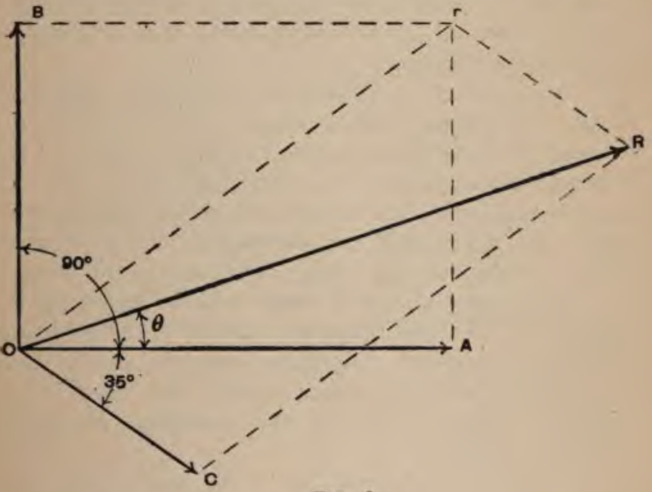

FIG. 8.

(10) **Mean Power of an Alternating Current.**—In article 7 (p. 20), the two full line curves in Fig. 4 were supposed to represent two E.M.F.'s, and it was shown how these could be *added together* to produce the resultant curve E_2. Let us now consider, not two E.M.F.'s differing in phase, but an E.M.F. and a current, also differing in phase;

and let us *multiply* one by the other in order to obtain the *power* in the circuit.

From what has been said on the subject of phase differences generally, the reader should have no difficulty in understanding that, whereas in the case of continuous currents the power in a circuit is given by the product of terminal potential difference and total current, this is very rarely true in the case of alternating currents.

If we connect a voltmeter across the terminals of an alternating-current circuit, and multiply the reading on this voltmeter by the actual value of the current in amperes, we shall obtain a number representing what are sometimes called the *apparent watts*, but which will not necessarily be a measure of the power actually being supplied to the circuit. The true power at any moment will be given by the product of the instantaneous values of current and impressed potential difference, and the mean value of all such products, taken during the time of one complete period, will be the quantity which we require to know.

In Fig. 9 the power, or watt curve has been drawn; it is obtained by multiplying together the corresponding ordinates of the curves of impressed potential difference E and of the current C.

Since the current lags behind the potential difference, it follows that, during certain portions of the complete period, the simultaneous values of E and C will be of opposite sign; that is to say, the current will be flowing *against* the impressed E.M.F.: the work done will therefore be *negative*, and these ordinates of the watt curve will have to be plotted *below* the datum line. This negative work (which is equal to the area of the shaded curve below the datum line) may sometimes almost equal the *positive* amount of work done, in which case the current is practically *wattless*—*i.e.*, the amount of energy put into the circuit during one quarter period is given back again during the succeeding quarter period.

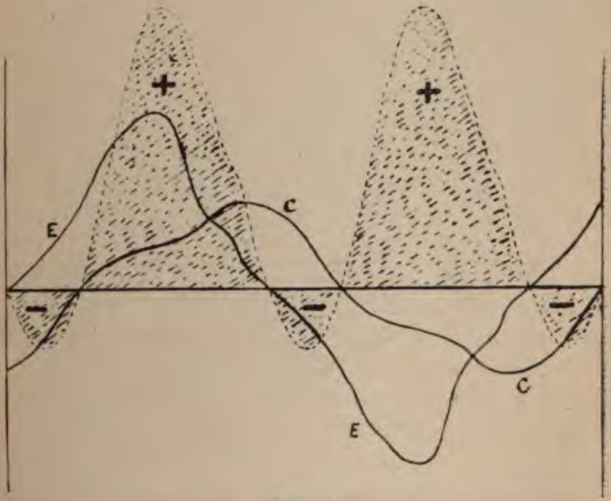

FIG. 9.

(The proper understanding of this state of things must of necessity present some difficulties to those unacquainted with alternating currents; but when dealing with the induction of an alternating-current circuit the matter will be again referred to.)

Returning to a consideration of the curves of Fig. 9, we see that the total amount of work done during one complete period is equal to the area of the two shaded curves marked +, *less* the area of the two shaded curves marked – ; and the mean *power* supplied to the terminals of the circuit will, therefore, be given us by the average ordinate of this dotted *watt* curve, due attention

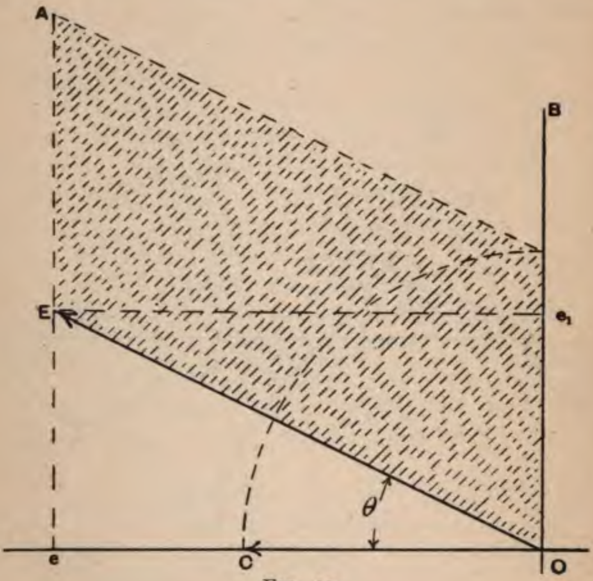

FIG. 10.

being paid to the *sign* of the instantaneous power values.

Instead of drawing a diagram such as Fig. 9, which is a very laborious undertaking, let us see what can be done with the simpler and more convenient vector diagrams

In Fig. 10 the $\sqrt{\text{mean square}}$ value of the

E.M.F. is represented by the vector $\overline{OE}$, and the $\sqrt{}$mean square value of the current by $\overline{OC}$.

The phase difference between these two alternating quantities—which, in Fig. 9, was indicated by the horizontal distance separating the zero or maximum values of the two curves—is now indicated by the angle θ; and since $\overline{OE}$ is in advance of $\overline{OC}$ (according to the clock-face analogy referred to above), it follows that the current C *lags behind* the E.M.F., E.

The method of compounding or adding together two or more E.M.F.'s was explained in article 9 (p. 24); and it follows from this that any vector, such as $\overline{OE}$, representing an E.M.F. can be considered as being made up of two or more component vectors.

Thus, in Fig. 10, if we draw through O the line O B at right angles to O C, and from the point E drop perpendiculars E e and E e_1 on to these two lines, we have in $\overline{Oe}$ and $\overline{Oe_1}$ two vectors representing imaginary E.M.F.'s which, when added together in the manner explained in article 9, would produce the resultant E.M.F., E.

Suppose, now, that there is no other E.M.F. in the circuit but that represented by the vector $\overline{Oe}$. This is in phase with the current, and the true watts are therefore obtained by multiplying together this E.M.F., e, and the current C*.

If, on the other hand, e_1 were the only E.M.F. in the circuit, the current C would be *wattless*—*i.e.*, the mean power in the circuit would be *nil*—because, as explained on p. 31, when the phase difference between the current and E.M.F. is 90 deg., the energy put into the circuit during one quarter of a period is given back again during the next quarter of a period, and the mean watts are, therefore, equal to zero.

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* See article 5, especially the concluding paragraphs (p. 16).

This leads us to the conclusion that, of the two (imaginary) components, e and e_1, of the total E.M.F., it is only the component e, *in phase with the current,* which need be taken into account when calculating the power in the circuit; and, given the angle of lag (or phase difference) between the applied potential difference E and current C, the total power in the circuit is obtained by projecting one of the vectors—such as O E—upon the other—O C—and then multiplying this latter quantity by the projection (O e) of the first one upon it.

In order to obtain a graphical representation of the power supplied to a circuit in which the current C lags behind the impressed volts E by an amount equal to the angle θ, we have simply to move round one of the vectors, let us say $\overline{\mathrm{O\,C}}$, through an angle of 90 deg., and then construct the parallelogram O A, the area of which will be a measure of the average value of the true watts; for it is evident that this area will always be equal to O C × O e.

(11) **Power Factor.** — Since e O E (Fig. 10) is a right-angled triangle, it follows that the power (C × e) supplied to the circuit, can be written C × E $\cos\theta$. But C × E is what we have already called the *apparent watts.* Hence the *real watts* = the *apparent watts* × $\cos\theta$, and this is the definition of the *angle of lag* between current and impressed E.M.F. which is the most useful. It is this multiplier ($\cos\theta$) which, since it was first suggested

by Dr. J. A. Fleming, is now generally referred to as the *power factor* of a circuit which is conveying an alternating current.*

As this expression occurs frequently in all literature dealing with alternating currents, it is important that the reader should have a clear understanding of its meaning.

In connection with continuous currents, the term is not used, for the simple reason that, once a steady flow of current has been established, the value of this current is always equal to the ratio of *applied E.M.F.* to ohmic resistance of the circuit; and the loss of power in the circuit may either be written C^2 R or E C.

In the case of alternating currents, it very often happens that the current is not *in phase* with the applied E.M.F., and the causes leading to this displacement of phase are sufficiently important to justify our devoting the following chapter to their consideration. The effects of such phase displacement between current and E.M.F. have, however, already been explained, and it will be understood that, although the power spent in heating up the conductors of the circuit may still be written C^2 R, the product E × C only represents the true power when the *power factor* is unity—*i.e.*, when the current is in phase with the applied E.M.F. Whenever there is the slightest phase displacement between the applied E.M.F. and the resulting current, the *apparent power* (E × C) has to be multiplied by the *power factor* (*cos* θ) in order to give us the true power in the circuit.

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* It has been assumed that the reader is not without at least an elementary knowledge of trigonometry; but should this assumption be unwarranted, it will suffice to point out that the *cosine* of the angle θ is the trigonometrical function of the angle, the numerical value of which is always obtained by dividing the length of the side O *e* of the right-angled triangle E O *e* by the length of the hypothenuse, O E. The expression $\cos\ \theta$ must, therefore, be understood to indicate the ratio of O *e* to O E, which ratio will, obviously, be constant for a given angle, and quite independent of the actual lengths O E and O *e*.

CHAPTER II.

SELF-INDUCTION AND CAPACITY.

(12) **Magnetic Field Due to a Continuous Current.**—Wherever there is a current of electricity there must, of necessity, be a corresponding magnetic condition of the surrounding medium.

In the case of continuous currents, once a steady value has been reached, the magnetic condition—*i.e.*, the number and direction of the magnetic lines—in the circuit remains unaltered. Thus, so long as the resultant exciting ampere-turns in a dynamo machine or direct-current motor remain constant, the total magnetic flux is maintained at a definite steady value. In order to calculate this total flux, it is necessary to know, not only the current and the number of turns which the circuit makes upon itself, but also the dimensions and configuration of the magnetic circuit which is linked with the electric circuit, and more especially the amount and disposition of any masses of iron in the direct path of the magnetic lines.

It is assumed that the reader has a fair working knowledge of the more important properties of the magnetic circuit; he should understand, for instance, the general lines on which the total magnetic flux in the armature of a dynamo can be approximately predetermined, for without a clear conception of the fundamental laws of the magnetic circuit, including the peculiar property of iron (and one or two other metals) of *increasing* the magnetic flux, the following arguments may not be readily understood.

Apart from the C^2 R losses in conductors, no work has to be done in order to *maintain* a magnetic field; but energy was spent in *creating* it, and this energy will all be given back again to the exciting circuit when the magnetic field is removed or withdrawn.

In order to form a mental picture of this property of an electric circuit, consider a flywheel, and neglect entirely all questions of bearing friction or windage—which, in our analogy, are equivalent to the C^2 R losses referred to above. Such a flywheel, once it has attained a definite speed of rotation, will continue to revolve for any length of time without requiring the further application of force. But a force had to be applied to bring it up to speed, and exactly the same amount of energy as was put into it is now available for doing work, and will be given back again by the time the flywheel has been brought to rest.

If we multiply the total flux in the core of a dynamo field magnet by the number of turns in the exciting coil, we obtain a quantity which is sometimes referred to as the *electromagnetic momentum* of the circuit. There is energy stored up in such a circuit, and if we attempt to open it suddenly, the results are frequently disastrous, because this energy *must* be dissipated in one form or another, and if we do not provide a short-circuiting resistance, or some means of drawing out the arc slowly, the dying down of the magnetism will induce such a high E.M.F. in the field coils as to break down the insulation or form a destructive arc at the point of disconnection. If, on the other hand, we dead short-circuit the winding, or connect its terminals—at the moment of switching off the supply—through an external resistance, the current in the coil, instead of being almost instantly interrupted, will die down in the manner shown on the diagram Fig. 11.

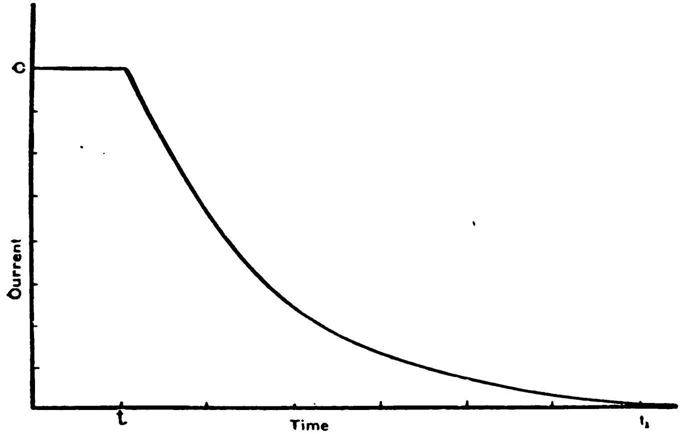

FIG. 11.

Here the horizontal distances from left to right represent lapse of time, and the vertical distances represent current. It will be noted that the current falls rapidly at first, but its rate of decrease diminishes as time goes on. Theoretically, the current never quite reaches zero value; but, in practice, it approximates to zero value in a very short space of time. In most cases, the whole distance from t to t_1 would only stand for a small fraction of a second; but in the case of a circuit with large *self-induction* such as the field coil of a dynamo, this time may extend over several seconds. The writer recollects receiving a shock from a large direct-driven two-pole dynamo, *after the same had been slowed down and come to a standstill*, at the moment of lifting the brushes off the commutator. This was entirely due to the current in the field coils —which were short-circuited through the armature— not having quite died down, and when the circuit was opened, the energy still stored in it took the form of a small current at a high potential. It should not be necessary to point out that, when the current dies down in the manner indicated in Fig. 11, the total energy put into the circuit when building up the field is given back in the form of C^2 R losses continuing for an appreciable *time* in the coil itself and the external resistance (if any) which is connected across the terminals.

The idea of the magnetic condition always existing simultaneously with the electric current must not be lost sight of. Thus, in Fig. 11, the reason why the current does not drop instantly to zero is briefly this: if it were to do so, the magnetic flux in the core (apart from the *residual* magnetism) would of necessity have to drop likewise instantaneously; but as, in so doing, it would induce a back E.M.F. of infinite value, and such as would tend to produce a current opposing the withdrawal of the magnetism, it follows that the current (and magnetism) in a circuit of appreciable resistance will actually die down in the manner indicated in Fig. 11; every decrease in the current (and magnetism) producing a certain *E.M.F. of self-induction* in the coil, tending to oppose a more rapid rate of decrease.

(13) **Magnetic Field Due to Alternating Current.**—When an alternating current flows in a circuit, the magnetic condition of the surrounding medium will vary in amount and direction in accordance with the variations of the current. For any given circuit, the amount of the magnetic flux may be approximately predetermined for any instantaneous value of the current, provided we know the length and cross section, or the *magnetic resistance* of the various parts of the magnetic circuit, and the number of turns in the electric circuit.* If there is no iron or other *magnetic* material in the magnetic circuit, the rise and fall of the magnetism will synchronise with the rise and fall of the current, and not only its maximum value, but also every intermediate value will be exactly proportional to the strength of the current. When the current reaches zero value, the magnetism will also pass through zero value; and when the current reverses in direction, the magnetic flux will reverse likewise.

Thus, if we plot the magnetism corresponding to every value of the current in a given circuit *containing no iron* (or other magnetic metal), we obtain such a curve as that shown in Fig. 12, where C is the current wave and M the magnetism. Every ordinate of this latter curve is equal to the corresponding ordinate of the curve C multiplied by a constant, and the magnetism is, of course, exactly *in phase* with the current.

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* If there is iron in the magnetic circuit, the effects of *hysteresis* must be taken into account. It should not be necessary to remind the reader that when carrying a piece of iron through a complete cycle of magnetisation, the ampere turns corresponding to a definite value of the induction on the rising portion of the cycle, are not equal to those corresponding to the same value of the induction on the descending portion, *i.e.*, when the current—after having passed through its maximum value—is being reduced.

In Fig. 13 the curve M is such as would be

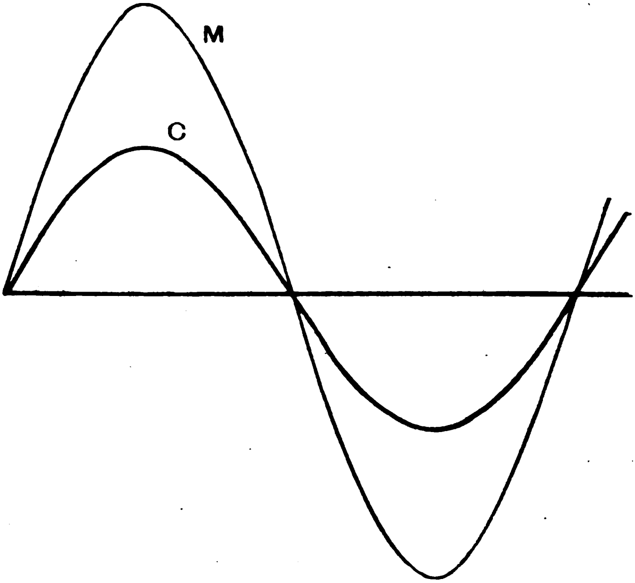

FIG. 12.

obtained if the magnetic circuit contained a large amount of iron. Here, with the one exception that the maximum value of the induction coincides with the maximum value of the current, it will be noticed that, on the whole, the magnetism *lags behind* the current.*

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* Hence the word *hysteresis* originally suggested by Prof. Ewing.

A full discussion of the various effects of *hysteresis*, or even a detailed description of the manner in which the curve M in Fig. 13 is derived from the curve C, hardly enters into the scope of the present book; but the reader requires only an elementary knowledge of magnetic phenomena—especially as regards the effects of iron in a magnetic field — to understand that the *residual magnetism*, which requires a reversed magnetising force to entirely eliminate it, is responsible for the peculiar relation of the two curves under consideration.

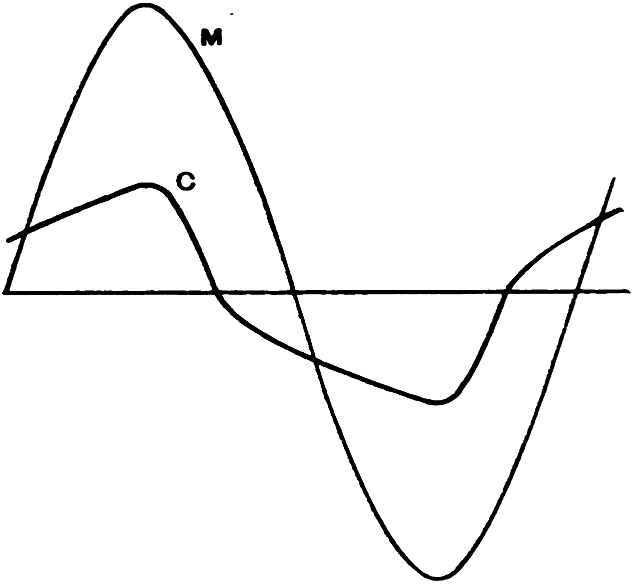

FIG. 13.

(14) **Self-Induction.** — The self-induction of a circuit, as, indeed, is sufficiently clearly indicated by the expression itself, is the induction or total number of magnetic lines *due to the current flowing in the circuit.* For any particular current C in the wire or conductor forming the electric circuit, the total magnetic flux of induction linked with this circuit *which is due to the current* C, is the *self-induction* corresponding to that particular current.

What is known as the *coefficient of self-induction* is a multiplier—generally denoted by the letter L—which takes into account not only the amount of the self-induction when unit current is flowing, but also the number of times this induction is threaded through the circuit. This, as we shall see presently, is important when considering the back E.M.F. due to self-induction; but as L is only constant in the case of a circuit containing no iron, the use of such a multiplier is limited in practice. It is, however, very largely used by those writers who prefer the analytical treatment of alternating-current problems, and for this reason, a sufficiently clear explanation of the exact meaning of the term will be found in Note 1 at the end of this book.

(15) **E.M.F. Produced by an Alternating Magnetic Field.** — We know that the E.M.F. generated in a conductor passing through a magnetic field is directly proportional to the rate at which the conductor is cutting through the (imaginary) magnetic lines—or tubes—of induction. If 100,000,000 C.G.S. magnetic lines are cut during one second of time, the resulting E.M.F. will be 1 volt. If we thrust a magnet into a coil of wire, the E.M.F. generated in the coil is proportional

to the strength of the magnet and the rapidity with which it has been thrust into the coil. The *direction* of this E.M.F. is always such as to tend to produce a current which will *oppose* the alteration in the magnetic condition.* If, on inserting the magnet, a positive E.M.F. was generated, a negative E.M.F. will be induced when the magnet is withdrawn.

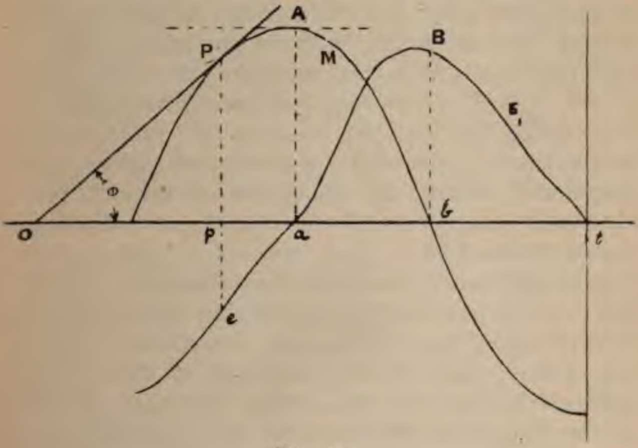

FIG. 14.

In Fig. 14, let the curve M represent the rise and fall of the alternating magnetism due to an alternating electric current—which has not been drawn in the diagram, but which, if there is no iron in the circuit, will be in phase with M, and if there is iron in the circuit, will be *in advance* of M (see Figs. 12 and 13). Let us see what is the E.M.F. which this varying magnetism will induce in the circuit. In the first place, it is evident that, if the magnetism were of a constant value — however large — no E.M.F. would be generated in the circuit. In other words, if its *rate of change* were *nil*, the value of the induced E.M.F. would be zero. This is exactly what occurs at the point A, when the curve M has reached its maximum value; the amount of magnetism is neither increasing nor decreasing at this particular instant, and consequently there can be no E.M.F. of self-induction. We can, therefore, plot the point *a* of the E.M.F. curve on the horizontal datum line immediately below the maximum value of the magnetism curve — this maximum value being graphically defined by the point of contact of the horizontal tangent to the curve M, as indicated by the dotted line.

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* There is no exception to this rule, which is known as Lenz's law.

Consider, now, the point *b* on the curve M. This point does not only represent the instant of time corresponding to the reversal in direction of the magnetism; but, in this example, it also indicates that portion of the cycle where the *rate of change* in the magnetism is greatest—as evidenced by the *slope* of the curve being steepest at this point. If this change is at the rate of 100,000,000 C.G.S. magnetic lines withdrawn from the circuit *per second*, an E.M.F. of 1 volt will be generated in every turn of the electric circuit which is linked with this magnetism.

Thus, if we know the number of turns in the circuit and also the scale to which the curve M has been drawn, we can readily calculate the actual induced E.M.F. at the instant *b* and plot this value *b* B to any convenient scale.

As to the *sign* of this E.M.F., it will, at this point, be *positive* (and therefore must be plotted *above* the datum line) because, since the magnetism is falling from a positive maximum towards a negative maximum, it is a *positive* E.M.F. which is necessary to produce a current such as would *oppose* or *counteract* this decrease in the magnetism.

We have seen that, where the curve M is horizontal, the induced volts are *nil*; if it were possible for this curve to drop so rapidly as to become perpendicular to the horizontal datum line, this would indicate an *instantaneous* change in the magnetic condition, or an infinitely great *rate of change*, and, therefore, an infinitely great induced E.M.F. whatever might be the actual number of magnetic lines added to or withdrawn from the circuit; and if the reader has a clear understanding of these extreme conditions he will probably accept the statement that the *steepness* or *slope* of the curve M is, at every point, an exact measure of the rate of change in the magnetic condition.*

As an example, suppose the ordinate p P represents 100,000 C.G.S. lines of magnetism. Draw the tangent O P to the curve M at the point P, and measure O p: let us assume that this distance corresponds to the two-hundredth part of a second. The increase in the magnetism at the point P is, therefore, at the *rate* of 100,000 C.G.S. lines in the two-hundredth part of a second, or twenty million lines per second; and if we assume that there are one hundred turns of wire in the circuit, the instantaneous value of the induced E.M.F. at the moment p in the hundred turns of wire will be

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* It would be an easy matter to prove this statement, and in all probability the reader will be quite capable of doing this to his own satisfaction. The author's purpose is not to prove every statement made in the course of this and subsequent chapters; but a general explanation, suggestive of the lines on which more exact proofs may be sought, will, wherever possible, be given.

$$\frac{20{,}000{,}000 \times 100}{100{,}000{,}000} = 20 \text{ volts,}$$

and this value, represented by $p\ e$, must be plotted *below* the datum line, because this E.M.F. will be such as will tend to produce a current in a *negative* direction—*i.e.*, such as would *oppose* the variation in the magnetic flux, which, at this moment, is *increasing* in amount.

The circuit we have been considering might be the armature winding of an alternator or the secondary coils of a transformer, and, in either case, the induced E.M.F., as shown in Fig. 14, will be a quarter period — or 90 deg. — *in advance* of the alternating magnetism to which it owes its existence; and, moreover, since exactly the same total magnetic flux as is threaded through the coils during one half-period is withdrawn during the succeeding half-period, the *mean* value of the positive half-wave of the induced E.M.F. is always exactly equal to the mean value of the negative half-wave.*

If N is the total amount of magnetic flux produced by the maximum value of the alternating current passing through the coil, and if S stands for the number of turns in the coil and n for the frequency, then the *mean* value of the induced E.M.F. is proportional to $N\,S\,n$, and this relation holds good whatever may be the shape of the current wave producing the magnetism. (See Note 2 at end of book.)

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* The *time integrals* of the positive and negative half-waves are equal, whatever may be the law of variation of the alternating magnetism.

It must not be supposed that because the *mean* value of the E.M.F. is independent of the wave form, the $\sqrt{\text{mean square}}$ value is likewise unaffected by the shape of the wave: on the contrary, the $\sqrt{\text{mean square}}$ value bears no definite relation to the mean value, but depends largely upon the wave form, which it is very difficult to predetermine with accuracy.

(16) **Reactance.**—In connection with continuous currents, the E.M.F. in the circuit can be expressed in terms of the current and resistance by writing Ohm's law in the form $E = C \times R$. We have just seen how there is another E.M.F., called the E.M.F. of self-induction, which very frequently asserts its presence in a circuit carrying an alternating current, and the term *reactance* has, for convenience, been given to the ratio obtained by dividing the E.M.F. of self-induction by the current in the circuit. Thus we may write

$$\textit{E.M.F. of self-induction} = \textit{current} \times \textit{reactance},$$

the last term being a multiplier which will be directly proportional to the *self-induction* of the circuit and to the frequency of the current, but which will also depend, to a certain extent, upon the wave form of the alternating current.

Thus, if a circuit has large reactance, the induced E.M.F., for a given current, will be greater than if the reactance is small. A load of incandescent lamps is an example of a circuit of small reactance, whereas the primary winding of a transformer—

which, in addition to enclosing a large magnetic flux, has also a considerable number of turns—is a circuit of large reactance; but this expression has *no meaning* except in relation to a circuit carrying an alternating current at a definite *frequency*. In fact, the chief reason why any reference has been made to this term is that it is used by many writers when treating of alternating currents, and without an elementary knowledge of the meaning of such terms, the reader would be seriously handicapped when taking up more advanced works on the subject.*

(17) **Current Flow in Circuit Having Negligible Self-Induction.**—Let us consider an electric circuit which is practically without self-induction, or electrostatic capacity. It may consist of a wire doubled back upon itself (in the manner adopted in winding resistance coils for testing purposes), or of glow lamps, or of a water resistance.

If an alternating E.M.F. is applied to the terminals of such a circuit, the current at any instant will be equal to the quotient of the instantaneous value of the E.M.F., divided by the total resistance of the circuit; or,

$$c_i = \frac{e_i}{R},$$

from which we see that the current wave will be of the same shape as the E.M.F. wave, and *in phase* with it—a state of things which is the evident result of the fulfilment of Ohm's law: for there is no reason for supposing that Ohm's law is not equally applicable to variable as to steady currents; it is only necessary to bear in mind that, in the case of variable currents, the *applied* E.M.F. and the effective or *resultant* E.M.F. in the circuit (to which the current is due) are not necessarily one and the same thing. In the case under consideration, of a circuit supposed to be without self-induction or capacity,* there is only one E.M.F. tending to produce a flow of current—*i.e.*, the E.M.F. supplied at the terminals of the generator: the current will therefore rise and fall in exact synchronism with the applied E.M.F.

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* The word *inductance* is used by some writers to express what we have called the *reactance* of a circuit; but since the same word is sometimes used to denote what has already been referred to as the *coefficient of self-induction*, the author has thought it advisable to avoid the use of it altogether, and so prevent confusion.

(18) **Current Flow in Circuit Having Appreciable Self-Induction.**—In order to get a better understanding of the whole question of self-induction in connection with alternating currents, let us consider an alternating current flowing in a circuit which has both ohmic resistance and reactance, as, for instance, a coil of wire of many turns, which, for the present, we will assume, has no iron core. Such a current is shown graphically by the curve C in Fig. 15, where intervals of time are measured, as usual, horizontally from left to right. The magnetism due to the current C will vary in amount and direction in accordance with the variations of the current. It may be calculated in the usual way for any given value of C, provided we know the length and cross-section, or the *magnetic resistance* of the various parts of the magnetic circuit, and the number of turns of wire in the coil. Let the curve *m* represent the rise and fall of this magnetism.

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* The effects of capacity will be dealt with in due course.

Since the *induced* or *back* E.M.F. due to these variations in the magnetic induction will be proportional to the *rate of change* in the total number of magnetic lines threaded through the circuit, we shall have no difficulty in drawing the curve E_1 (as in article 15, p. 45), which represent the E.M.F. of self-induction, and which lags exactly one-quarter of a period behind the current wave.

We are now in a position to determine the potential difference which must exist at the terminals of the circuit in question, in order that the current C will flow through it.

Draw the curve E_2 to represent the E.M.F. required to overcome the ohmic resistance. It will be in phase with the current, because its value at any point is simply C × R, where R stands for the resistance of the circuit. Now add the ordinates of E_2 to those of an imaginary curve exactly similar but opposite to E_1, and the resulting curve E will evidently be that of the impressed potential difference which, if maintained at the ends of the circuit under consideration, will cause the current C to flow in it. Thus we see how the relation between the impressed E.M.F. and the resulting current may be graphically worked out for any given case.

From a study of the curves in Fig. 15 it is evident that the effect of self-induction is to make the current lag behind the impressed E.M.F. If the E.M.F. required to force the current against the ohmic resistance is small in comparison with the induced E.M.F., the lag will be very considerable; it cannot, however, exceed one quarter of a complete period, which limit is only reached when the E.M.F. of self-induction is so large, and the ohmic resistance of the circuit so small, as to render the E.M.F. required to overcome this resistance of no account.

In order to briefly sum up the principles governing the flow of an alternating current in a circuit having self-induction, we may say that the varying current produces changes of magnetism, which again produce a varying E.M.F., called the E.M.F. of self-induction. This, together with the E.M.F. already existing (and without which no current would flow), produces the *effective* or *resultant* E.M.F. By dividing the value of this resultant E.M.F. at any instant by the total ohmic resistance of the circuit, the corresponding current intensity is obtained. This condition *must* always be fulfilled, otherwise Ohm's law would not be satisfied

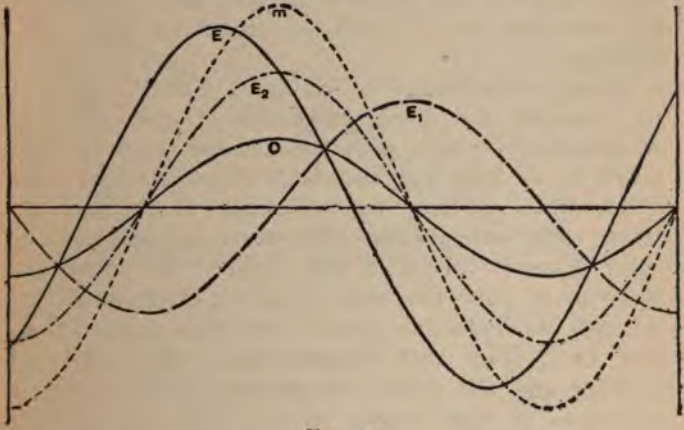

FIG. 15.

(19) **Diagram Showing Relation of E.M.F.'s in Inductive Circuit.**—We have seen in article 15, p. 44, and again in discussing the curves of Fig. 15, that the induced E.M.F. lags exactly 90 deg. behind the magnetism — or self-induction; and since this magnetism rises and falls in synchronism with the current to which it owes its origin, the same relation—*i.e.*, a phase difference of a quarter period—exists between the induced E.M.F. and current in an inductive circuit, such as we have been considering.

Again, the effective or resultant E.M.F. required to overcome the ohmic resistance of the circuit is necessarily in phase with the current (refer to the curve E_2 in Fig. 15), and both these relations hold good, whatever may be the shape of the current wave, provided always that there is no iron or other *magnetic* metal in the circuit. If, therefore, we wish to draw the vector diagram of the E.M.F.'s in such an inductive circuit, the two vectors $\overline{O\,E_1}$ and $\overline{O\,E_2}$ (Fig. 16)*, representing respectively the *induced* and the useful or *effective* E.M.F.'s, must be drawn at right angles to each other, with $\overline{O\,E_2}$ in *advance* of $\overline{O\,E_1}$.

The necessary impressed E.M.F. at the terminals of the circuit is found—as explained on p 52, article 18—by compounding the E.M.F. E_2 with an imaginary E.M.F. (*o e*) exactly equal, but *opposite in phase* to E_1. In this manner the vector $\overline{O\,E}$ is obtained.

Now complete the triangle $E\,O\,E_2$, and note that the lengths of the sides of such a triangle represent spectively the effective or useful E.M.F.; the back M.F. or E.M.F. of self-induction; and the total pressed E.M.F. at the terminals of the circuit. oreover, since the side O E is the hypothenuse a right-angled triangle, this latter quantity (the pressed E.M.F) is always equal to the square root the sum of the squares of the other two, which an easy and useful rule to bear in mind.

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* Compare with Fig. 10, p. 32.

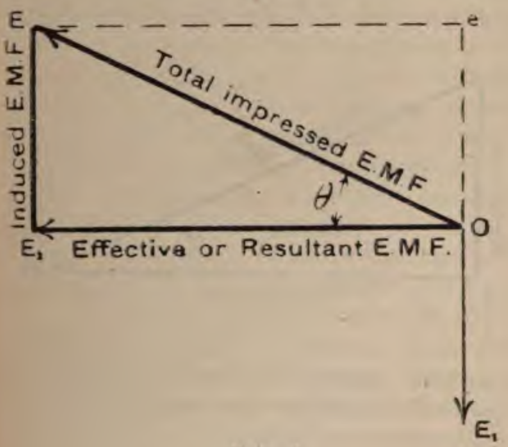

FIG. 16.

It should not be necessary to remind the reader that the angle θ is the *angle of lag* between the effective and the impressed volts, and that *cos* θ, or the ratio $OE_2 \div OE$ is the *power factor* of this particular circuit.

In calculating the various E.M.F.'s in the circuit, as drawn in Fig. 16, the value of the current has necessarily been taken into account. Let us eliminate this factor entirely, by dividing the three quantities O E_1, O E_2, and E E_2 by the value of the current. The triangle need not be altered in any way; it has merely to be considered as having been drawn to a different scale. It is reproduced in Fig. 17, and it will be noted that the line O E_2 now stands for the *resistance* of the circuit, while E E_2 represents the *reactance* (see article 16, p. 49). With regard to O E, since this is the result obtained by dividing the total E.M.F. by the current, it evidently stands for the *apparent resistance* of the circuit, to which the name *impedance* has been given.

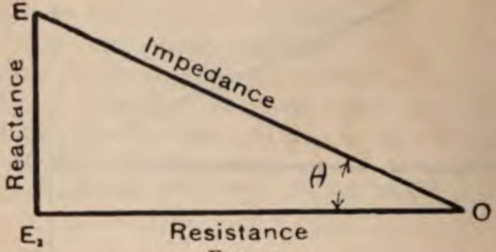

FIG. 17.

We may, therefore, write:

$$impedance = \frac{impressed\ volts}{amperes};$$

and again,

$$impedance = \sqrt{(resistance)^2 + (reactance)^2}.$$

It must not be overlooked that such expressions and relations as these have only a limited usefulness, and the quantity understood by the word *impedance* is merely a multiplier, which is not constant, even for a given circuit, but which depends—among other things—upon the frequency and wave form of the impressed E.M.F.

A little consideration will make it clear that the higher the periodicity, the more important becomes the *reactance* relatively to the *resistance*, and that, in all cases where the back E.M.F. is objectionable, it is advisable to keep the periodicity as low as possible.

(20) **Effect of Iron in Magnetic Circuit.** — Although it is not intended to investigate the causes which lead to a distortion of the current wave, and a certain loss of energy when iron is introduced in the magnetic circuit, it will, nevertheless, be necessary to briefly note the effects produced when iron is used —as in generators, motors, and transformers—for the purpose of increasing the magnetic induction and concentrating it at certain points.*

There are two causes of loss of energy when the path of the magnetic lines is through iron ; the first is due to eddy currents, and the second to hysteresis.†

(21) **Eddy Currents.** — Whenever a conductor of electricity is placed in a fluctuating magnetic field, Foucault or eddy currents are produced. These eddy currents are in phase with the induced E.M.F., and they will tend to oppose the changes in the magnetism. Other conditions being similar, the mean demagnetising tendency of such currents will be directly proportional to the specific conductivity of the metal in the path of the magnetic lines ; the power necessary to maintain these currents is

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* The reader is referred to the author's book, "Alternating Currents, and the Theory of Transformers," for a simple explanation of the most important effects due to the presence of iron in the magnetic circuit.

† See p. 41.

dissipated in the form of heat in the mass of the metal, and this loss of power would be very considerable unless special precautions were taken to reduce it.

The remedy—which is adopted in the cores of all alternating-current machinery—is to laminate or divide the metal core in a direction parallel to the magnetic flux, and slightly insulate the adjacent plates or wires from each other. In this manner the losses may be reduced to a very small amount.

(22) **Hysteresis.** — The losses due to hysteresis occur only in the case of the magnetic lines passing through a *magnetic* metal, such as iron, and they are in no wise dependent upon the degree of lamination of the magnetic circuit.

It is well known that all iron, even the softest and purest, retains some magnetism after the magnetising force has been removed. By applying a magnetising force in the opposite direction this *residual* magnetism is destroyed, and the magnitude of this force—or, in other words, the amount of work which has to be done to withdraw this magnetism—depends upon the quality of the iron. Soft annealed wrought iron retains most magnetism; but, on the other hand, it parts with it more easily than the harder qualities of iron and steel, and for this reason requires the least expenditure of energy to carry it through a given cycle of magnetisation.

The frequency, or number of alternations of the magnetism per second, does not appreciably influence the loss through hysteresis *per cycle*, and, therefore, the loss of power per cubic inch or per pound of iron in the path of the magnetic lines will be proportional to the frequency. It has also been ascertained experimentally that the energy expended in carrying a given sample of iron through one complete cycle of magnetisation is approximately proportional to the 1·6th power of the limiting induction; and if B stands for the maximum value of the induction, we may, therefore, write

watts lost per pound of iron $\propto$ $B^{1·6}$ × *frequency.*

In the case of well-annealed commercial iron stampings, the hysteresis loss per pound at a frequency of 50 periods per second would be about ·09 watts with a limiting induction of 15,000 C.G.S. lines *per square inch* of cross-sectional area, and about ·25 watts at double this induction—*i.e.*, 30,000 C.G.S. lines.

Assuming the iron stampings to be ·014 in. thick, and sufficiently well insulated from each other, the eddy-current losses for the same frequency and maximum inductions would be approximately ·02 and ·07 watts per pound; from which it will be seen that the losses due to eddy currents in the iron cores are of less importance than the hysteresis losses, and, indeed, with low inductions and low frequencies the eddy-current losses are almost negligible.

(23) **General Conclusions Regarding the Introduction of Iron in the Magnetic Circuit.**—Let us consider a choking coil consisting of a number of turns of wire, and draw two vector diagrams for such a circuit: the first (Fig. 18) showing relation of current to E.M.F. on the assumption that the solenoid or coil of wire has an air core—*i.e.*, no iron core—and the second (Fig. 19) showing what takes place in the same coil when an iron core is introduced for the purpose of reducing the current.

For the sake of comparison, we shall assume, in the first place, that the resistance, R, of the coil is constant in both cases, and equal to *four* ohms; and in the second place we shall see what are the relations of impressed E.M.F. and current to induce in each case a constant back E.M.F. of 100 volts.*

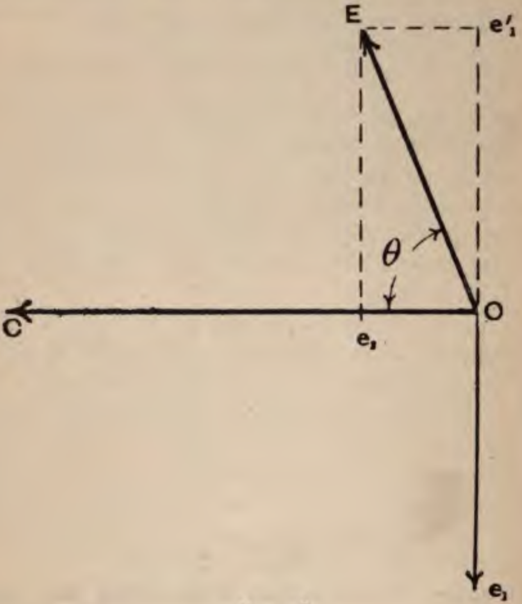

FIG. 18.

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* The relation between magnetic flux and current was discussed in art. 13; and in art. 15 it was explained how the induced E.M.F. can be calculated for any particular point in the cycle of magnetisation. We shall, therefore, assume that the current required, in the present example, to produce the given induced E.M.F. of 100 volts, can be readily calculated; but if the reader wishes to go more fully into the question, he is referred to Note 2 at the end of the book.

n Fig. 18 (which should be compared with . 10, p. 32) draw $\overline{O e_1}$ of such a length as to resent the induced E.M.F. of 100 volts, and $\overline{O C}$

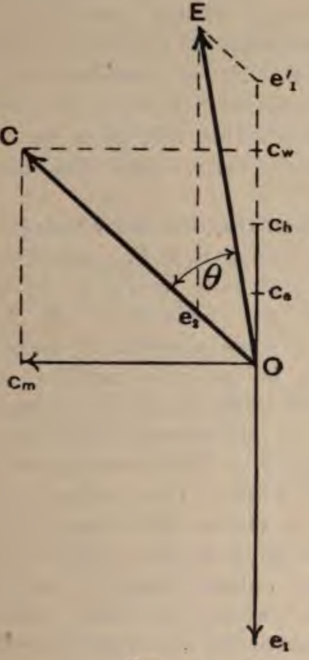

Fig. 19.

ctly 90 deg. *in advance*, to represent the gnetising current, that is to say, the alternating rent of which the maximum value will produce necessary magnetic flux to induce the E.M.F. of volts in the windings of the coil. We shall pose this current, C, to be exactly 10 amperes.

The E.M.F., e_2, required to send this current against the resistance of 4 ohms will be

$$e_2 = 10 \times 4 = 40 \text{ volts},$$

and this component of the total E.M.F. must be drawn *in phase* with $\overline{\text{O C}}$. The total necessary impressed E.M.F., E, is obtained by compounding $\overline{\text{O}\, e_2}$ and an imaginary E.M.F. exactly equal, but opposite to $\overline{\text{O}\, e_1}$. This will give us the value of E as about 108 volts, and the angle of lag $\theta = 68$ deg.

It will be noticed that the current lags considerably behind the impressed E.M.F., and this lag would approach more and more nearly to its limiting value of 90 deg. if the resistance of the coil were diminished: if it were possible to make $R = 0$, the current C would be actually *wattless*; as it is, the power lost in the choking coil is all spent in heating up the wire—the alternating magnetic flux costs nothing to produce. The actual power lost in our example is 400 watts. It is equal to $C^2 R$ or to $\text{O C} \times \text{O}\, e_2$, or to $\text{O C} \times \text{O E} \times \cos \theta$.

We shall now suppose that the magnetic circuit, instead of being entirely through air, is made up either partly or wholly of iron, which must be laminated to prevent an abnormal loss through eddy currents.

Draw $\text{O}\, e_1$ in Fig. 19, as before, to represent 100 volts; but $\overline{\text{O C}_m}$—*i.e.*, the necessary magnetising current—will not be so great as O C in Fig. 18, because the introduction of the iron core has provided an easier path for the magnetism. We shall suppose that $\overline{\text{O C}_m} = 5$ amperes, or half the magnetising current of the previous example. This is not, now, the *total* current in the coil, but merely that component of the total current which is required to produce the necessary magnetic flux: it must be drawn, as before, exactly 90 deg. in advance of $\mathrm{O}\,e_1$.

Let us suppose that the *eddy currents* in the iron are responsible for a loss of power equal to 150 watts, and that the *hysteresis* is responsible for 300 watts. This total power of 450 watts has to be supplied by the generator—or whatever source of supply the choking coil is connected to—in the form of a component of the total current flowing *against* the induced E.M.F., the portion of the current required to provide for the eddy-current loss being $\frac{150}{100} =$ 1·5 amperes, while the hysteresis component of the current will be $\frac{300}{100} = 3$ amperes.

These two components of the total current are shown as $\overline{\mathrm{O\,C}_e}$ and $\overline{\mathrm{O\,C}_h}$ in Fig. 19, and they are drawn in a direction exactly *opposite* to $\overline{\mathrm{O}\,e_1}$ (the induced E.M.F.). The total current $\overline{\mathrm{O\,C}}$ is obtained by compounding $\overline{\mathrm{O\,C}_m}$ and $\overline{\mathrm{O\,C}_w}$, the last vector being equal to the sum of the two *work* components, $\overline{\mathrm{O\,C}_e}$ and $\overline{\mathrm{O\,C}_h}$.

As regards the E.M.F., the volts required to overcome the ohmic resistance of the coil are, as before, in phase with the total current $\overline{\mathrm{O\,C}}$; and since this measures very nearly 7 amperes, the vector $\overline{\mathrm{O}\,e_2}$ must be drawn of such a length as to represent $7 \times 4 = 28$ volts.

The resultant or total necessary impressed E.M.F. is obtained by compounding $\overline{\mathrm{O}\,e_2}$ and the imaginary E.M.F. $\overline{\mathrm{O}\,e_1'}$ exactly equal and opposite to $\overline{\mathrm{O}\,e_1}$. It will be found that the angle of lag θ is now about

38 deg., while the total power lost = O C × O E $\cos \theta$ = about 650 watts.

Thus, although the C^2 R losses are less, owing to the reduction of current due to the introduction of an iron core, the losses in this core are themselves of such importance as to make the total losses greater than in the previous example. This, however, is by no means necessarily the case when iron is introduced in the magnetic circuit. The assumptions made for the purpose of drawing the above diagrams are arbitrary, but they serve the purpose of showing in what manner the losses in the iron core appear in the supply circuit and affect the magnitude and phase displacement of the total current.

(24) **Capacity.**—The effects of electrostatic capacity are of far more importance in alternate - current working than in connection with continuous currents, and we shall briefly examine the conditions arising out of the introduction of a condenser in a circuit carrying an alternating current.

The chief difference, in this respect, between a continuous — or unidirectional — and an alternating current is that the former, after charging up the condenser, will be absolutely interrupted, while the latter will continue to flow to an extent depending upon the capacity of the condenser, the pressure, and the frequency (not to mention the wave form, which must also be taken into account).

Any arrangement of two conductors of electricity separated by an insulator forms a condenser, of which the capacity will be large or small depending upon the nearness, or otherwise, of the conductors, and the nature of the dielectric between them.

Not only will a concentric cable have appreciable capacity, owing to the arrangement and proximity of the inner and outer conductors, but overhead conductors will also have a certain capacity relatively to each other, and relatively to the earth.

The effects of electrostatic capacity in the case of a conductor supported on poles at a reasonable distance from the ground are very small, even on a long transmission line, *provided the frequency is low;* but further reference will be made to this particular case when dealing with the transmission of power by polyphase currents. The capacity of a concentric underground cable, especially when insulated with rubber, is a much more serious matter. The capacity per mile of any particular make and size of cable will always be furnished by the manufacturer, and the total capacity of any feeder or distributing system can, therefore, be readily calculated when the length of the different sizes of cable is known.

(25) **Current Flow Through a Condenser.** — If a couple of ballistic galvanometers are connected to the two coatings of a condenser, in series with a battery or other source of continuous E.M.F., as shown in Fig. 20, it will be found, on depressing the key K, that the swing of both galvanometers is such as to indicate that the same quantity of electricity which passed into one set of plates has passed out of the other set of plates. But, nevertheless, the condenser is now *charged*, and if the key K_1 be depressed (after releasing the key K), the same quantity of electricity will flow through the circuit as when the charging key was depressed, only the flow will be in an opposite direction, as indicated by the swing of both galvanometers being approxi-

mately the same as before, but in the reverse direction.*

Bearing these facts in mind, it will not be difficult to understand that an alternating current will have the property of flowing through a condenser, and the following remarks should assist the reader in forming a mental picture of what takes place.

In Fig. 21, A is an alternator and K a condenser, which, in practice, might be a long concentric cable with the two conductors disconnected and well insulated at the distant end.

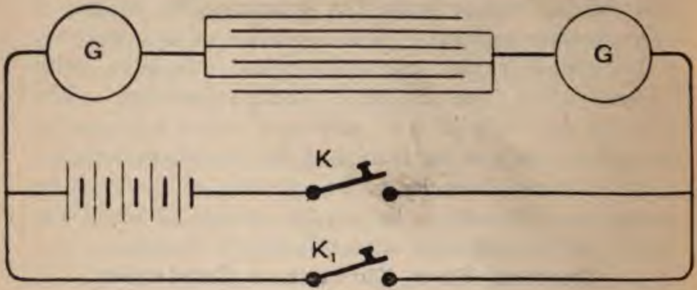

FIG. 20.

So long as the alternating current is flowing in a positive direction, the condenser is being charged, but as soon as ever the current reverses, the condenser begins to discharge. The maximum charge of the condenser—which corresponds with the maximum value of the condenser E.M.F.—will, therefore, of necessity occur at the instant when the current is *changing in direction.*

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* A charged condenser should be considered, not as containing a store of electricity, but a store of electric energy, which may be used for doing useful work by joining the terminals of the condenser through an electric circuit. In this respect it may with advantage be compared with a deflected spring which, so long as it is kept in a state of strain, has a certain capacity for doing work. (See also Note 3 at end of book.)

In Fig. 22 the curve c represents the current flowing through the condenser, and the dotted curve q represents the charge (in coulombs) which, it will be noticed, has been steadily growing from its *negative* maximum value to its *positive* maximum value (d) during the whole time the current has been flowing in a positive direction. This curve q will therefore lag behind the current curve by exactly a quarter of a period.

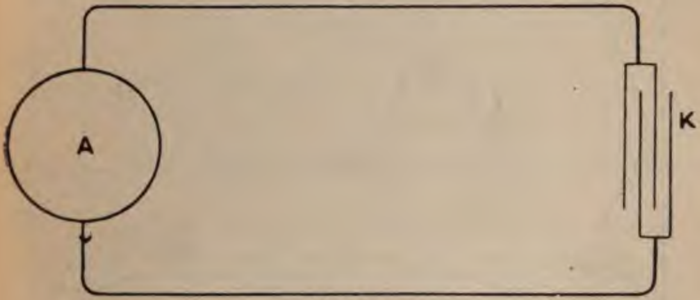

FIG. 21.

Next, as regards the condenser E.M.F.; it is evident that this will always be such as to *oppose* the applied E.M.F., or, in other words, it will tend to *expel* the charge.

We may, therefore, draw the curve e_1 exactly opposite to q, the length of the ordinate at every point being equal to

$$\frac{\textit{quantity (or charge of electricity)}}{\textit{capacity of condenser}}.$$

If we neglect the resistance of generator and leads, and also certain losses which occur in the condenser itself, the curve e_1 in Fig. 22—*i.e.*, the potential difference at condenser terminals—will be exactly equal and opposite to the alternator E.M.F. Thus, the *condenser E.M.F.* is exactly a quarter of a period *in advance* of the current; and this is interesting when it is remembered that the *E.M.F. of self-induction* always *lags behind* the current by the same interval.

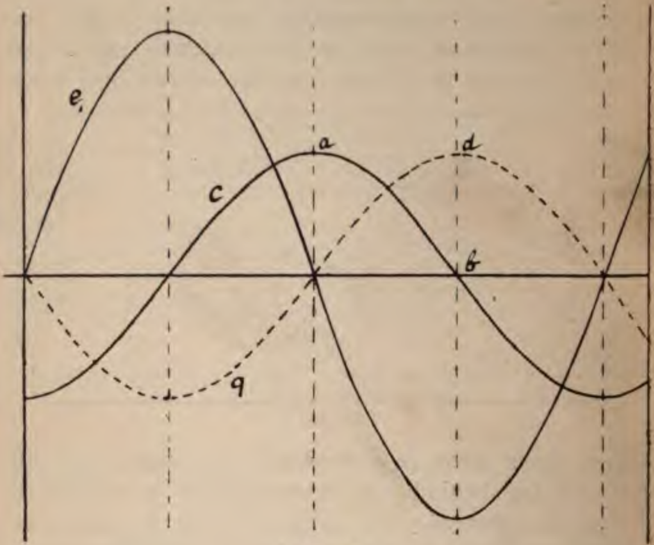

FIG. 22.

This suggests the possibility of counteracting the effects of induction by the introduction of capacity, or *vice versa*, and, indeed, the one is sometimes used in practice for the purpose of partially neutralising the effect of the other, with moderate success; variations in the frequency or any slight modification in the wave form of the applied E.M.F., such as will occur as the load on the generator is varied, will considerably interfere with the balancing action of a condenser and choking coil.

As regards the magnitude of the condenser current (c in Fig. 22), *if we assume the E.M.F. wave to be a sine curve**, it can be shown that

$$c = \frac{2 \pi n \text{ K E}}{1{,}000{,}000}$$

where n = the periodicity;
K = the capacity in microfarads;
E = the applied E.M.F. at condenser terminals in volts.

In very long concentric feeders, the current flowing into the cable when there is no connection at the distant end is sometimes quite appreciable, especially when the cables are insulated with rubber, and the frequency and pressure are high. This is the capacity current, and its effect in increasing the total current is readily ascertained when it is remembered that this component of the total current will be 90 deg. out of phase with the useful current delivered at the distant end of the feeder, if we assume the load to be non-inductive.

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* It is important to note that the capacity current taken by a cable, or other condenser, will depend considerably upon the wave form of the applied potential difference. Thus, in place of the multiplier 2π (or 6·283), in the formula for a simple harmonic variation—or sine wave—we should have to substitute $4\sqrt{3}$ (or 6·928) if the wave were of a triangular shape, with straight sides, and this corresponds to an increase of 10 per cent. in the amount of the capacity current. A peaked form of wave is, therefore, to be avoided if it is desirable to keep down the capacity current; but since the wave form of all alternators changes, to a certain extent, as the load comes on, there will, in any case, be a variation in the capacity current depending upon the load, and this variation may even amount to as much as 50 per cent. It is, however, the capacity current at *light loads* which is of the greatest importance in practice, because at times of heavy load its effects are not felt to anything like the same extent. Should the E.M.F. wave have several peaks—*i.e.*, more than one maximum value per half-wave—this would be quite the worst form of wave as regards the capacity current, which might easily be doubled, solely on account of this peculiarity in the shape of the wave.

Thus, if

C = the ingoing current;
C_1 = the condenser current (say, 10 amperes);
C_2 = the outgoing current (say, 60 amperes),

we may write

$$C = \sqrt{C_1^2 + C_2^2} = 60{\cdot}82 \text{ amperes},$$

which is very little greater than the outgoing current; and the example illustrates the fact of the capacity current becoming of less and less importance as the load on the feeder increases.

Perhaps one of the most extensive systems of concentric, rubber-insulated, underground cables is that of the Societé Anonyme Belge Eclairage Electrique de Saint Petersbourg. There are about 240 kilometres of cable in St. Petersburg forming the high-tension network of this company's mains. The charging current, for the frequency of 42·5 and a pressure of 2,000 volts, is about 47 amperes; but there are nearly 1,000 transformers on the circuits, taking a total magnetising current of about 250 amperes, which more than balances the abnormally large capacity current.

CHAPTER III.

POLYPHASE CURRENTS — GENERAL PRINCIPLES, AND SYNCHRONOUS GENERATORS.

(26) When single-phase alternating currents were first used, on a large scale, for incandescent electric lighting, no satisfactory single-phase motors were obtainable. One of the difficulties in the way of producing commercial alternating-current motors was, no doubt, the high frequency (about 100 in this country) which had been adopted for the lighting circuits ; but, apart from this, the problem of designing single-phase motors capable of starting under load, is by no means a simple one. The introduction of polyphase currents, by the aid of which a *rotating magnetic field* could be obtained from stationary windings, completely solved the problem of the alternating-current motor.

The idea of producing a rotating magnetic field by means of polyphase currents may be said to date as far back as 1879, when Walter Baily read a paper before the Physical Society, entitled "A Mode of Producing Arago's Rotation" ; but, although several patents were taken out during the ensuing years, nothing serious was done in this connection until 1891, when M. von Dolivo-Dobrowolsky and Mr. C. E. L. Brown put down their complete scheme of three-phase power transmission from Lauffen to

Frankfurt. After this, the use of polyphase motors extended rapidly.

Definition.—A polyphase current may be defined as a combination of two or more alternating currents, all of the same periodicity, but having certain phase differences between them.

(27) **Production of Rotary Field.**—In Fig. 23 we have a representation of a two-phase current, the phase angle of which is 90 deg. It will be seen that this diagram merely represents two simple harmonic alternating currents, A and B, of the same frequency, and the same magnitude, but differing in phase by a quarter of a period.

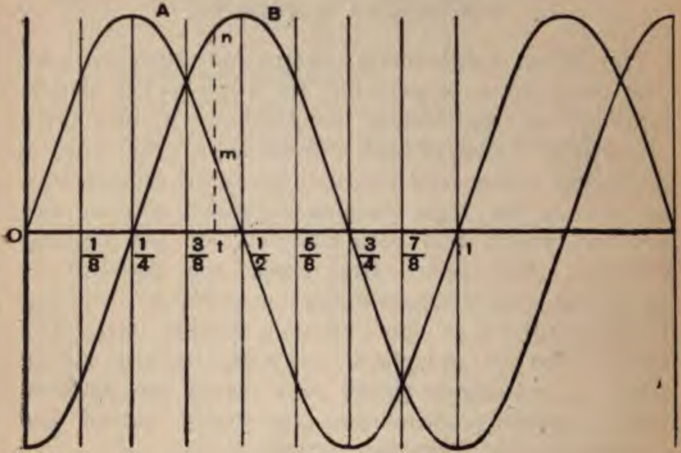

FIG. 23.

Let us imagine these two currents to be flowing respectively in two coils, A and B, set at an angle of 90 deg. to each other, in the manner shown in

Fig. 24. We shall further assume that when the current A is flowing in a positive direction—*i.e.*, when it is measured *above* the datum line, Fig. 23—it will produce a magnetic field in the upward direction through the coil A; and when the current B

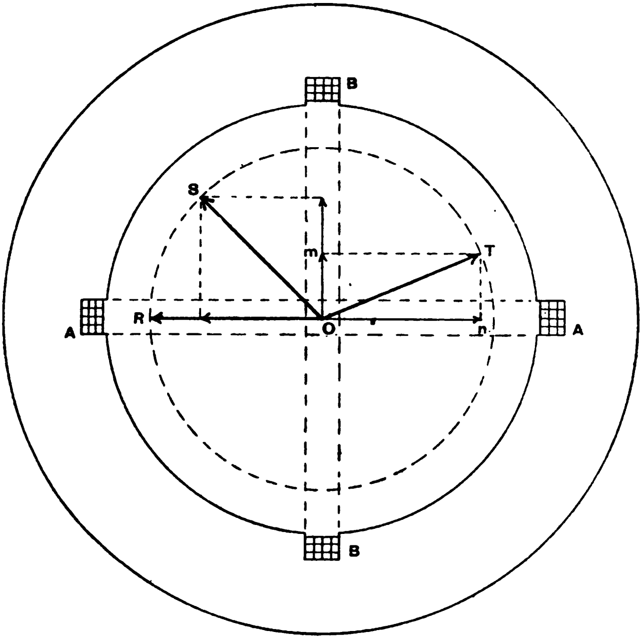

FIG. 24.

is positive, it will produce a field from left to right through the coil B.

In order to see clearly how the resultant field, due to the currents in the two coils, revolves round the centre, O (Fig. 24), let us start at the instant O

(Fig. 23). The resultant field at this instant is evidently $\overline{O\,R}$, which represents the field due to the maximum negative value of the current B—the value of A, at this instant, being zero.

After the lapse of one-eighth of a period, the two currents (on the assumption of a sine curve wave form) are exactly equal, but A will be positive and B negative: the resultant field will be $\overline{O\,S}$. After another eighth of a period, the resultant field will be that due to the maximum positive value of A; it would be represented by a vector equal in length to O R, but drawn in a vertical direction from the centre, O, upwards. At the instant t (corresponding to the 7/16ths of a complete period) $A = \overline{t\,m}$ and $B = \overline{t\,n}$, both being positive. The combination of these two component fields gives us $\overline{O\,T}$ as the resultant. It is not necessary to carry the construction any further, for it is evident that the combined effect of the two currents will be to produce a magnetic field of constant intensity, equal to that due to the maximum value of either current, revolving round O as a centre at the uniform rate of one revolution in the time of one complete period.

Although we have assumed a phase difference of a quarter period between the two currents, and an angle of 90 deg. between the axes of the two coils, it should be stated that a rotating field can be obtained by means of two phase currents, whatever may be the phase difference between them.

The necessary condition for the production of a uniformly rotating field of constant strength by the combination of two alternating magnetic fields is simply that the angle which the positive directions of the alternating fields make with each other must be the *supplement* of the phase angle.

Thus, if we had assumed a phase difference of 100 deg. instead of 90 deg. between the two currents shown in Fig. 23, we should have found it necessary to draw the coils A and B in Fig. 24 with an angle of (180—100) deg. or 80 deg. between them instead of at right angles to each other.

With regard to the intensity of the resulting rotating field, on the assumption of the sine law of variation, this will be equal to the maximum value of either of the two (equal) alternating fields, *multiplied by the sine of the phase angle.*

Although the proof of the above statements is simple, we shall content ourselves with an illustration, which, at the same time, will explain the use of another diagram, sometimes preferable to the curves of Fig. 23, for representing alternating quantities which follow the simple harmonic law of variation.

We shall assume the phase difference between the two currents to be one-third of a period, instead of one-quarter of a period, as in the previous example.

Draw $\overline{\text{O A}}$ and $\overline{\text{O B}}$, in Fig. 25, to represent the *maximum values* of the two alternating magnetic fields. These vectors must subtend an angle of 120 deg. between them to represent the phase difference of one-third of a period.

With O as a centre, describe the dotted circle of radius equal to O A or O B, and on each end of the vertical diameter describe a smaller circle, of radius equal to half O A or O B, in the manner shown in the figure.

Now imagine the two vectors to revolve in a *counter-clockwise direction*—as indicated by the arrow—at a uniform rate of one revolution in the time of one complete period. The length of either vector, measured from the centre, O, to the point where it

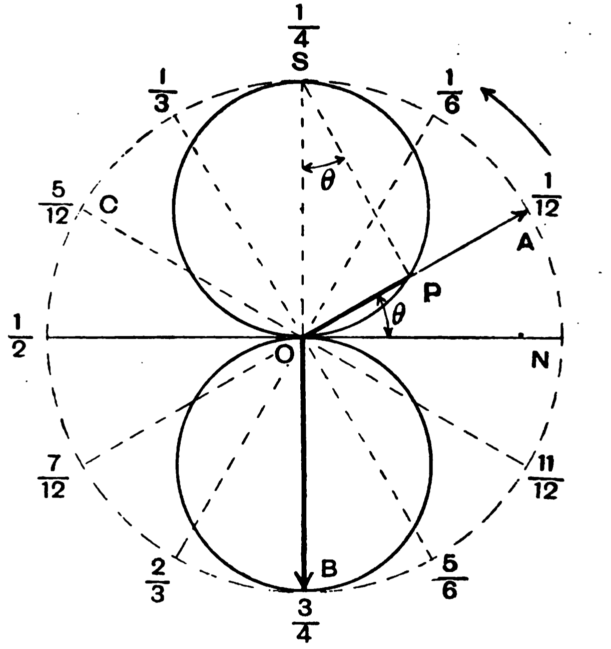

FIG. 25.

is cut by one of the inner circles will then be a measure of the field intensity due to the current in phase A, or in phase B, as the case may be, at any particular instant throughout the complete cycle. Measurements made in the upper circle would denote, say, a positive direction of the magnetic field, while measurements made in the lower circle would indicate a magnetic flux in the opposite, or negative, direction.

The proof of the above statement is very simple. In the first place, the angle O P S of the triangle O P S, constructed within a semicircle of which O S is the diameter, is always a right angle; and since the angle P S O is equal to A O N, we see that O P is always equal to O S $\sin \theta$, or O A $\sin \theta$, which is the same thing. Now, the reason why the simple harmonic curves as shown in Fig. 23 are also called sine curves, is simply because the ordinates of such curves are directly proportional to the sine of the time angle; and it, therefore, follows that the portion, O P, of any revolving vector such as $\overline{OA}$ or $\overline{OB}$, which is contained within either of the two small circles, will be a measure of the instantaneous value of an alternating quantity which follows the simple harmonic law of variation.

Let us now see how the combination of the two alternating fields, A and B—when B lags behind A by one-third of a period (120 deg.)—will produce a rotating field when combined at an angle of (180—120) deg. or 60 deg.

In Fig. 26, the two coils are shown at an angle of 60 deg. to each other; and, since the field due to each coil will be at right angles to the plane of the coil, the field component due to coil A will always lie on the diameter 10—4, while the field component due to B will lie on 12—6.

At the particular instant corresponding to the position of the vectors shown in Fig. 25 the resultant field will be $\overline{O-1}$, obtained by compounding $\overline{O\,c}$ and $\overline{O\,d}$. After an interval of a twelfth of a period (30 deg.), the two components will be equal, but of the same signs as before—*i.e.*, A will still be positive, and B negative: the resultant field is $\overline{O-2}$ of which the components are $\overline{O-12}$ and $\overline{O-4}$. After another twelfth of a period, the vector A (Fig. 25) will have reached its maximum positive *value,* while B will have fallen to half its maximum

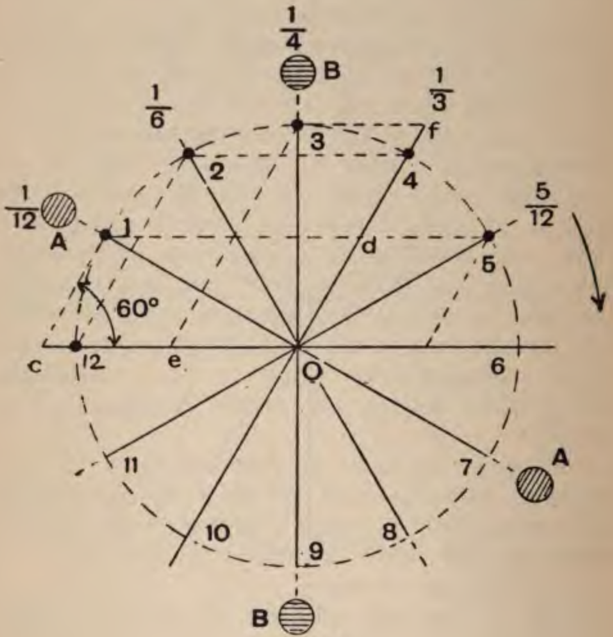

Fig. 26.

negative value: the resultant field will be $\overline{O-3}$, obtained by compounding $\overline{O\,e}$ and $\overline{O\,f}$. And so on: it will be found that the resultant field will revolve round O as a centre, at a uniform rate of one revolution per period, and it will have a constant value represented by the radius of the dotted circle in Fig. 26. This radius is evidently equal to O *c* sine 60 deg.—that is to say, the intensity of the uniform rotary field will—for this particular phase difference—be equal to the maximum value of any one of the component fields multiplied by the sine of 60 deg.

This confirms the general statement previously made, to the effect that the resulting rotary field is equal to the maximum value of either of the two equal alternating fields, multiplied by the *sine of the phase angle*. It is true that the phase angle, in our example, is 120 deg., and not 60 deg., but the sine of 120 deg. is the same as sine (180—120) deg. or 60 deg.

Three-Phase Currents.—It has been shown how a rotating field may be produced by means of two-phase currents, and the reader should, therefore, have no difficulty in understanding that the same results can be obtained by using any number of currents, with suitable phase differences between them, and a corresponding number of field coils arranged to subtend the proper angles between each other.

Fig. 27 shows the three currents of a three-phase system with a phase angle between them of 120 deg., or one-third of a complete period. It is not proposed to draw any more diagrams on the lines of Figs. 25 and 26; but it will be understood that we have merely to imagine a third vector drawn in Fig. 25 in the direction O C, and a third coil in Fig. 26 across the diameter 11—5, when it will be found that the combination of the three alternating fields will produce a uniform rotating field, exactly as in the case of the two-phase system; the only difference being that the intensity of the rotating field will

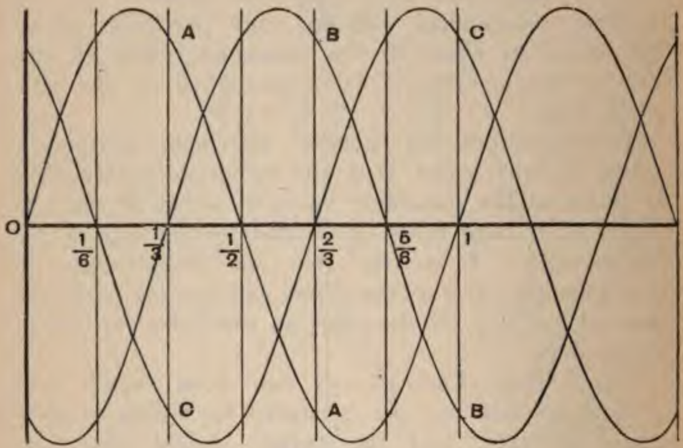

FIG 27.

now be equal to 1·5 times the amplitude of any one of the three (equal) alternating fields.

In practice the rotating field is not necessarily of constant strength, neither does it always revolve at a uniform rate. Thus, if the currents in the coils do not follow the sine law, the resultant field will vary in amount.

Again, if the induction in the iron forming the magnetic circuit be carried to high values, the *permeability* may be far from constant, and, even with a current following the simple harmonic law, the magnetism might vary in a very different manner. It is, therefore, advisable to work with comparatively low inductions, and to use generators giving, as nearly as possible, sine curve E.M.F. waves under all conditions of load.

The effect of inaccurate spacing of the coils, or of unequal currents in the different phases, will also be to produce distortion of the resulting field; but the remedy is obvious.

(28) **Utilisation of Rotary Field.**—Consider a small continuous-current dynamo, of which the field magnets are fully excited, but with the brushes lifted off the commutator.

If there is no fault in the armature winding, it will be possible—by means of a lever or pulley fixed to the shaft—to revolve the armature by hand; because, although E.M.F.'s will be generated in the windings, these balance each other, and there will be no circulating current.

If, on the other hand, one or more of the armature coils are short-circuited, the effect will be the same as if a strong brake had been applied, and it will not be possible to revolve the armature by hand, except at a very slow rate. If only one coil be short-circuited, it will be possible to move the armature freely so long as the shorted conductors lie in the gap between the poles; but as soon as they enter the fringe of magnetism under the pole tips, the braking effect will be felt. Again, if all the coils are shorted—a result which may be attained by winding a few turns of bare copper wire round the commutator—there will be no positions of the armature between which it can be freely moved; but the magnetic braking action will be felt uniformly throughout the complete revolution of the armature.

It is evident that these effects are due to the currents generated in the closed coils of the armature, for these will always flow in such a direction as to oppose the motion of the conductors through the field. The torque which has to be exerted to make the armature revolve, will be proportional to the product of field strength and armature ampere turns under the pole-pieces; and the power required to revolve the armature will be expended in the form of $C^2 R$ losses in the armature windings.

Let us now suppose that the whole framework of the dynamo, including the system of field magnets, instead of being bolted to solid foundations, is free to revolve on the armature spindle. We shall not — with this arrangement — experience the same difficulty in revolving the armature, because—if the system of (excited) field magnets be perfectly balanced, and the friction of the bearings be small—the revolving armature will draw the field magnets round with it; and, indeed, if frictional losses could be entirely neglected, the armature and field would be magnetically locked together, while the whole system could be revolved in space without requiring any expenditure of power.

If there is appreciable frictional resistance to the motion of the system of field magnets, the latter will revolve at a slightly slower speed than the armature; that is to say, there will be a certain *slip*, or *relative speed* between the two parts, which will automatically adjust itself until the currents generated in the armature conductors are sufficient to produce the necessary torque.

Although we have briefly considered the case of a revolving armature dragging the field with it, the arrangement can be reversed, and if the system of field magnets be revolved, the short-circuited armature will be dragged round at approximately the same speed: the E.M.F.'s generated in the armature windings, for a constant strength of field, will depend upon the *rate at which the magnetic lines are cut by the armature conductors*—*i.e.*, upon the *relative* speed of field and armature.

In a polyphase motor, the armature, or *rotor*, may consist of three or more windings *closed upon themselves*, or—as is frequently the case in small machines—it may be of the squirrel-cage type, in which heavy conductors, threaded through the iron stampings near the periphery, are joined together, at each end, by substantial metal rings. In both cases the effect is the same: the revolving field — produced in the manner already described, by the windings on the *stator*—generates E.M.F.'s in the short-circuited rotor windings, and the resulting current produces a torque tending to drag the armature round against the resisting forces; these resisting forces being made up of bearing friction, windage, hysteresis, and eddy-current losses when running light, with the addition of the torque due to the load on the motor when the latter is doing useful work.

Although the theory of the polyphase motor will be treated more fully in a later chapter, it is important that the reader should have a clear mental picture of the revolving field dragging the short-circuited armature with it. Imagine the resistance of the closed armature windings to be *nil*; then, whatever may be the load on the motor, its speed would be absolutely constant; the armature would be magnetically locked with the revolving field, because the slightest "slip," or difference in speed between field and armature, would induce infinitely great currents in the windings of the latter, and instantly pull it into step.

In practice the *slip* does not exceed 5 per cent., and it will reach its greatest value when the motor is doing its maximum load. It may be asked, what does this 5 per cent. difference of speed represent? The answer is that it represents power lost in heating the armature conductors. Thus, if the field revolves at 1,000 revolutions per minute and the armature at 960 revolutions, and if the torque at this particular speed is 100 statical foot-pounds, we have:

Horse-power *imparted to armature*

$$= \frac{100 \times 2\pi \times 1{,}000}{33{,}000}$$
$$= 19{\cdot}05$$

Horse-power *developed by armature*

$$= \frac{100 \times 2\pi \times 960}{33{,}000}$$
$$= 18{\cdot}3$$

hence, horse-power wasted in $C^2 R$ losses in armature conductors

$$= {\cdot}75$$
$$= 560 \text{ watts.}$$

A thorough understanding of how the current in the primary circuit (stator windings) grows in response to the demand for additional power as the load on the rotor increases, is not possible without a knowledge of the main principles underlying the working of alternate-current transformers: we shall, therefore, leave the more detailed consideration of the induction motor for the present, and briefly deal with the question of polyphase generators.

(29) **Polyphase Generators.**—It is not intended, in these pages, to give formulæ or data which would be of use to those engaged in the design of polyphase generators; but an attempt will be made to give a clear explanation of the elementary principles involved in the generation of two, or three-phase currents.

In nearly all large alternators or polyphase generators, the system of field magnets revolves, while the armature is stationary. With this arrangement only two collecting rings are required, for conveying the exciting current to the field coils; the armature current being delivered from stationary terminals. The insulation of these terminals, together with that of the armature coils, is more easily carried out than if they formed part of the movable portion of the machine.

For low pressures, such as 200 volts, especially if the output is small, a design of machine with revolving armature will generally be found to be cheap and efficient.

A very mechanical form of generator is that known as the inductor type, in which *all* the windings are stationary, no collecting rings being required: a portion of the magnetic circuit revolves, and, as this is provided with polar projections, the necessary variation of the magnetic flux through the armature coils is readily produced. The polar projections, in this type of machine, must be laminated to prevent losses through eddy currents.

Nearly all alternating-current generators are of the multipolar type—that is to say, they are provided with more than one pair of poles. A few machines, when driven at high speeds by steam-turbines, may have only two poles, but these are the exception.

If p is the number of poles, and R is the speed in revolutions per minute, then

$$\textit{Frequency} = \frac{R}{60} \times \frac{p}{2}$$

and in the case of large machines, running at comparatively low speeds, a fairly large number of poles will be found necessary to give the desired frequency.

Whatever may be the type of machine, or number of poles, we may consider the armature conductors to be cut by the magnetic lines in the manner indicated in Fig. 28. Here we have a diagrammatic representation of single-phase, two-phase, and three-phase windings. In each case, the system of alternate pole-pieces is supposed to move across the armature conductors in the direction indicated by the arrow. It will be noted that the conductors of each phase are shown connected up to form a simple wave winding; but this is only done to simplify the diagram, and it will be readily understood that each coil may contain a number of turns, attention being paid to the manner of its connection to the succeeding coil, in order that the E.M.F.'s generated in the various coils shall not oppose each other.

The upper diagram shows a single winding, in which an alternating E.M.F. will be generated. In the middle diagram there are two distinct windings, A and B, so arranged relatively to each other and

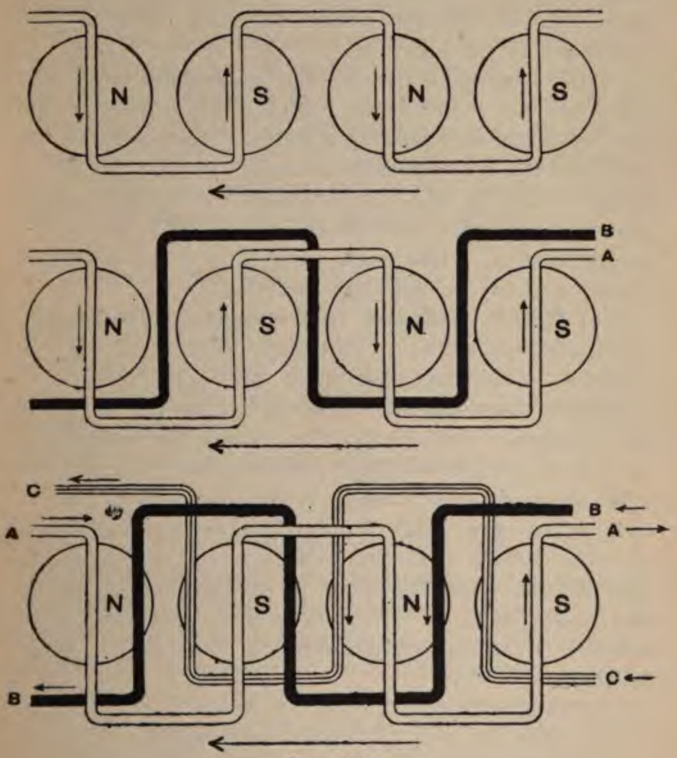

FIG. 28.

the pole-pieces that the complete cycle of E.M.F. variations induced in A will also be induced in B, but after an interval of time represented by a quarter of a period. The diagram shows the positions of the poles at the instant when the E.M.F. in A is at its maximum, while in B it is passing through zero value. From these two windings we can, therefore, obtain two-phase currents with a phase difference of 90 deg.

In the bottom diagram, the arrangement of three windings is shown from which three-phase currents can be obtained, having a phase angle of 120 deg. or one-third of a period between them. It will be noticed that, at the instant indicated by the relative positions of coils and poles, the E.M.F. in A is at its maximum, while in B and C it is of a smaller value, and in the opposite direction.

(30) **E.M.F. Induced in Generator Windings.**—With the assistance of the top diagram of Fig. 28, it will be readily seen that the total number of magnetic lines cut by any one armature conductor *per revolution* will be N × p, where N stands for the total magnetic flux passing into the armature from any one pole, and p is the number of poles.

If S be the total number of active conductors, counted round the circumference of the armature, and R stands for the speed in revolutions per minute, the *mean* E.M.F. in volts generated in any one winding if all the conductors are connected in series, will be

$$E_m = Np \times S \times \frac{R}{60} \times \frac{1}{100{,}000{,}000} \quad . \quad . \quad *$$

We are not generally concerned with the *mean* value of an alternating quantity; but it is not always possible to estimate very accurately the *shape* of the induced E.M.F. wave, which will determine the relation of the $\sqrt{\text{mean square}}$ value of this E.M.F. to its mean value. This will depend upon the disposition of the windings, the shape and spacing of the poles, etc. In order to obtain the actual volts at terminals on open circuit, we shall have to include a multiplier, k, which will depend upon the *form* of the wave. This multiplier, which is the ratio of the $\sqrt{\text{mean square}}$ value to the mean value of the induced E.M.F., is sometimes called the *form factor*. The general expression for the open-circuit terminal pressure therefore becomes:

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* See art. 15, Chap. II.

$$E = k \, N \, p \, S \, \frac{R}{60} \times 10^{-8}$$

If the E.M.F. wave is a sine curve, $k = 1 \cdot 1$.

If the wave were rectangular in shape (which would not be possible in practice), the multiplier k would be unity.

Other calculated values of k are:

For triangular shape $k = 1 \cdot 16$

For semi-circle …… $k = 1 \cdot 063$.

(31) **Connections of Polyphase Armature Windings.**—Consider the armature winding of an ordinary continuous-current two-pole dynamo. If we imagine the commutator of such a machine to be entirely removed, the winding—whether the armature be of the drum or ring type—will be continuous, and closed upon itself. If the armature be revolved between the poles of separately excited field magnets, there will be no circulating current in the windings, because the magnetism which passes out of the armature core induces an E.M.F. in the

conductors exactly opposite and equal in amount to that induced by the entering magnetism.

If we now connect to a couple of slip rings two points of the winding from the opposite ends of a diameter, the machine will be capable of delivering an alternating current. If we provide three slip rings, and connect them respectively to three points on the armature winding distant from each other by 120 deg., the machine will become a three-phase generator.

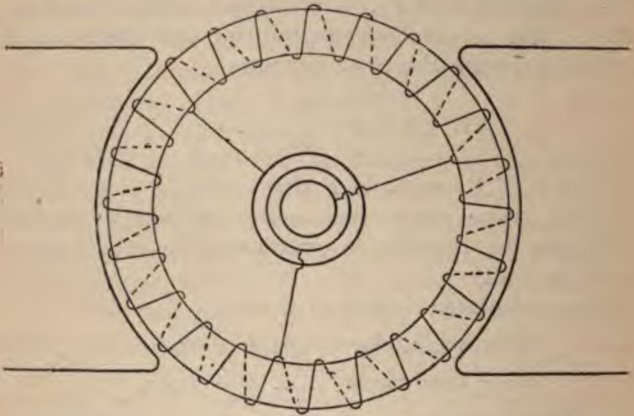

FIG. 29.

In this manner, polyphase currents of any number of phases can be obtained, and if the windings, polar spaces, etc., are symmetrical, there will be no circulating current.

This method of connecting up the various armature coils of a polyphase generator is known as the *mesh* connection. In the case of three-phase currents it is also sometimes referred to as the *delta* connection.

The diagram, Fig. 29, shows the three equidistant tappings from armature winding to slip rings required to produce three-phase currents. It is evident that the potential difference between any two of the three rings will be the same, since each section of the winding has the same number of turns, and occupies the same space on the periphery of the armature core. Moreover, the variations in the induced E.M.F. will occur successively in the three sections at intervals corresponding to one-third of a complete period.

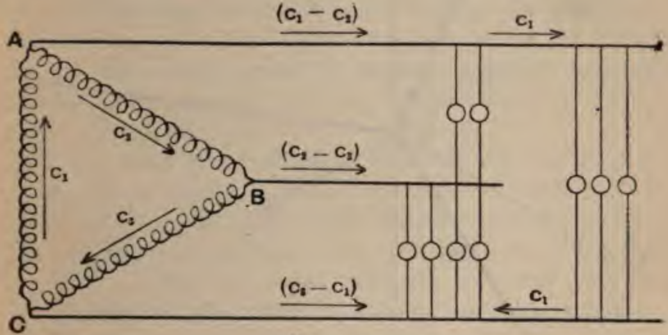

FIG. 30.

Incandescent lamps may be connected across one or all three phases as shown in Fig. 30. This lamp load is practically non-inductive; and we may, therefore, consider the current to be in phase with the potential difference across the lamp terminals.

Let the vectors $\overline{OC_1}$, $\overline{OC_2}$, and $\overline{OC_3}$, in Fig. 31, represent the loads on the three sections of the three-phase generator. They are drawn at an angle of 120 deg. to each other, because—being in phase with their respective E.M.F.'s—they must necessarily differ in phase by a third of a period.

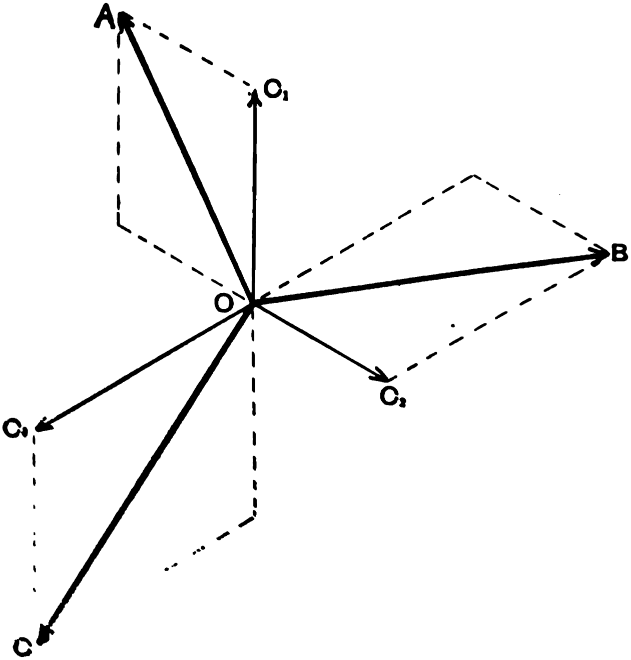

FIG. 31.

Referring to Fig. 30, we see that the resultant currents, at the three terminals of the generator, which flow into or return from the external circuit are:

At the terminal A .... $C_1—C_2$
" " B .... $C_2—C_3$
" " C .... $C_3—C_1$

It is, therefore, an easy matter to draw the vectors $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ in Fig. 31: these represent, by their length and phase relations, the currents in the three conductors leading to the lamp load.

It will be found that any one of these three vectors always balances the other two; that is to say, any one vector, such as B, will be found to be exactly equal but opposite in direction to the resultant of A and C. That this is necessarily the case is evident when it is realised that, at every instant, the total of all currents leaving the armature must be equal to the total of all currents returning to the armature, which is merely another way of saying that the sum of the three currents, A, B, and C, must be zero.

If the load is balanced, which will be the case if there is an equal number of lamps on each of the three sections, or if the load consists, not of lamps, but of induction motors, then the three currents, C_1, C_2, and C_3 will be equal, and the currents leaving the terminals will also be equal, but greater than the armature currents: it can easily be shown that—for this condition of a balanced load—any one of the line currents is equal to the current, C_1, in any one of the armature sections, multiplied by 2 cos 30 deg., or by $\sqrt{3}$. Thus

$$\begin{aligned} A &= C_1 \sqrt{3} \\ &= 1{\cdot}732\, C_1 \end{aligned}$$

Nature of Circulating Current in Practice.—Although there is, theoretically, no circulating current in the mesh-connected armature windings of a polyphase generator, the conditions of practice are such that perfect balance is rarely, if ever, attained, and currents will, therefore, circulate in the windings in addition to those delivered to the mains. Such currents do not necessarily involve any appreciable loss of efficiency, but they must evidently increase, to a certain extent, the $C^2 R$ losses in the armature conductors.

The circulating current with the field magnets fully excited, but with open external circuit, would be a small percentage of the normal full-load current in a well-designed machine; at the same time, if there is want of symmetry in the arrangement of armature coils, or in the spacing and shapes of the pole-pieces, this circulating current might amount in practice to as much as 25 per cent. of the full-load current. A little consideration will make it clear that the function of this internal armature current is to maintain the potential differences at the three terminals such that their sum is, at every instant, equal to zero. A study of such currents with the aid of the oscillograph might prove instructive; but it is clear that their direction at every instant will be such as to react upon the pole-pieces, and so produce the necessary correction in the distribution of the magnetic lines entering the armature. One interesting conclusion to be drawn from these considerations is that the amount of the circulating current will depend upon the strength of the field, or on the magnetic induction. In other words, if an ammeter were inserted in series with the closed armature windings, it would be found to indicate an approximately steady reading notwithstanding large variations in the *speed* of the machine, *provided the excitation of the field magnets remained constant.*

If the speed were kept constant, the amount of circulating current would be approximately proportional to the volts at terminals.

Star Connection of Three-Phase Armature Windings.—In practice this method sometimes has advantages over the mesh or delta connection. It consists merely in joining together the starting ends of the three armature windings in one common junction, or *neutral point*, and taking the finishing ends to the three terminals. It is not necessary to run a common return connection back from the load to this neutral point, because such a connection would not carry any current; each arm of the star connection forms the return path for the currents flowing in the other two arms.

This method of connecting up the coils will be again referred to when dealing with the question of transmission of power by three-phase currents; but if we refer back to Fig. 31 and imagine the vectors $\overline{O\,C_1}$, $\overline{O\,C_2}$, and $\overline{O\,C_3}$ to stand, not for the *currents* in a mesh-connected armature, but for the E.M.F.'s in the three sections of the star connection, then the vectors $\overline{O\,A}$, $\overline{O\,B}$, and $\overline{O\,C}$ will represent the potential differences between the three terminals of the generator.

A simpler construction is shown in Fig. 32. Here $\overline{o\,e_1}$, $\overline{o\,e_2}$, and $\overline{o\,e_3}$ represent the E.M.F.'s in the three sections of the star-connected armature, and the sides of the triangle obtained by joining the ends of these three vectors, correctly represent — both by their lengths and their directions—the amounts and phase relations of the pressures as measured between terminals.

If, as is usually the case, the three armature sections have the same number of turns, the triangle of vectors representing the terminal pressures will be equilateral and—as explained in connection with the resultant *current* of a *mesh*-connected armature—the *volts* measured across the terminals of a *star*-connected machine, will be $\sqrt{3}$ times greater than the volts measured across any one of the three (equal) armature sections.

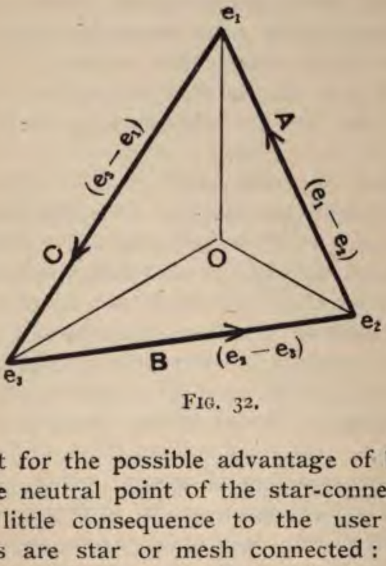

FIG. 32.

Except for the possible advantage of being able to earth the neutral point of the star-connected winding, it is of little consequence to the user whether his machines are star or mesh connected: he is merely concerned with the *volts at terminals*, and not with the combination of windings which produces this voltage.

Two-Phase Armature Connections.—It is sometimes advisable to keep the two circuits of a two-phase supply entirely separate, in which case the generator would be provided with four terminals and there would be four conductors connecting the machine to the load; each of the two circuits can be treated as a single-phase alternating-current supply, and loaded independently of the other; at the same time, if the two circuits are brought to an induction motor with stator coils correctly spaced, a revolving field will be produced.

Except when transmitting the current to long distances (in which case certain complications arise), a saving of copper may be effected by having one common return conductor which will carry a total current equal to the sum of the two outgoing currents.

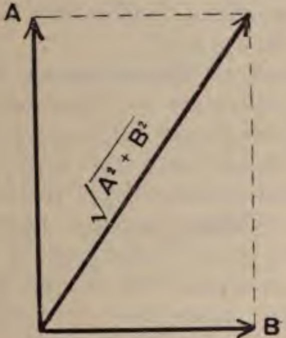

FIG. 33.

If the amounts of the two outgoing currents are respectively A and B, the return current will not be A + B, because the two component currents are not in phase; it will be equal to $\sqrt{A^2 + B^2}$ for a phase difference of 90 deg., as will be evident from a glance at Fig. 33.

Thus, if—as would be the case on a load consisting of induction motors—the currents are equal in the two phases, the return current, in the common conductor, will only be $\sqrt{2}$ times as great as the current in any one phase.

The equivalent to the delta or mesh connection of the three-phase generator, would be obtained by taking four tappings off the closed armature winding, instead of three as in Fig. 29. A generator with windings connected in this manner may, therefore, be considered indifferently as a two-phase or a four-phase machine.

(32) **Regulation of Synchronous Generators.**—The polyphase generators we have been considering belong to the class known as synchronous machines, to distinguish them from the asynchronous generators, introduced comparatively recently, and which are practically polyphase induction motors reversed—*i.e.*, of which the rotor is mechanically driven. This latter type of machine will be briefly referred to after the induction motor has been more fully dealt with

Since the synchronous polyphase generator is very similar to its prototype, the single-phase alternator, the scope of this book will only permit of the following points being treated somewhat superficially. At the same time, the manner in which the regulation of an alternator depends upon the nature of the external load is sufficiently important to claim our attention.

In a continuous-current dynamo, the drop in pressure, at constant speed, between no load and full load, is due not only to the resistance of the armature windings, but also, and more especially, to the armature reaction—*i.e.*, to the distorting and demagnetising effects of the armature current.

The same thing occurs in an alternator, and, if the load is non-inductive—that is to say, if the current is in phase with the potential difference at generator terminals—the machine will behave in a very similar way to a dynamo in respect to the question of voltage drop.

In Fig. 34, the alternate poles are supposed to be travelling at the back of the armature coils, from right to left. The rectangles A, B, and C represent three different instantaneous positions of an armature coil relatively to the poles. In position A the induced E.M.F. is at its maximum, and tends to send a current in the direction of the small arrows. In position B the E.M.F. is zero (since there is no change in the number of magnetic lines passing through the coil), while in position C the E.M.F. is again at its maximum, but in the opposite direction to what it was in position A.

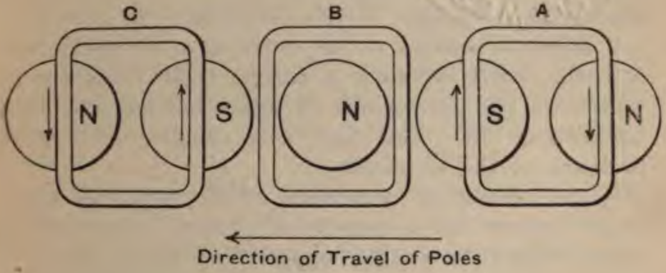

FIG. 34.

If the load is non-inductive, the current will be in phase with the E.M.F , and, although it will tend

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to *distort* the magnetic field, it will have no direct effect in either strengthening or weakening the inducing field as a whole. Now imagine the load to be inductive; the current will lag behind the E.M.F., and will, therefore, reach its maximum value when the pole-pieces have travelled beyond the position indicated at A. The result will be that the current in the armature coils will have a demagnetising effect on the field magnets. We have only to imagine the worst possible condition, of the current lagging 90 deg., to see that the poles will occupy the position B relatively to the armature coil *when the current in the latter is at its maximum*, and the effect of such a current will evidently be to *weaken* the pole which is opposite the coil, thus diminishing the total flux and causing a drop of pressure at the terminals.

If the current is in *advance* of the E.M.F.—which might occur on a circuit of large capacity—the effects would be exactly opposite to those obtained with a lagging current: the poles would be *strengthened*, and this strengthening effect might even be sufficient to çounteract the weakening due to distortion,* and the drop due to armature resistance; the result being that the peculiar nature of the load might alone be sufficient to maintain constant volts at the terminals of the machine.

(33) **Compounding Synchronous Generators.**—The foregoing remarks will make it clear that, in order to successfully compound an alternating-current generator, the increased field excitation must not

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* Which, by crowding the magnetic lines to one side of the pole-pieces, increases the magnetic resistance, and diminishes the total flux through the armature coil.

merely depend upon the amount of the armature ampere turns, but also upon the nature of the load—*i.e.*, on the power factor—and whether the armature current is leading or lagging. In practice the necessity will not arise of dealing with the question of heavy leading currents, although such currents will frequently occur, at *light loads*, on a circuit having large capacity, such as would be obtained with an extensive system of underground cables. The requirements of a perfect alternator, to give constant volts under all conditions of load, when driven at constant speed may be summed up as follows:

1. The terminal pressure should remain constant notwithstanding alterations in amount of the armature current or in the power factor (within practical limits).

2. The compensating windings or devices should be such as to be instantaneous in their operation, so that the generator may respond promptly to the altered conditions of load.

3. The compensating or compounding devices must not interfere with the successful running of the machines in parallel.

This last condition may be disposed of at once by stating that, in the case of compounded alternators, it may be just as necessary to provide an equalising bar and equalising circuits between the machines as when running direct-current compound-wound dynamos in parallel.

With regard to the condition 2, it is evident that this removes from the field of useful competition all automatic devices, the principle of which depends upon the variation of the field current by the cutting in or out of resistances. It should be pointed out, in this connection, that, even if the current round the field magnets were to alter instantly with the change of load, it might yet be a matter of several seconds before the terminal pressure could regain its correct value owing to the large self-induction of the field circuit.*

Probably the most successful compound generator on the market is the self-exciting synchronous machine devised by Mr. Alexander Heyland, whose work in connection with induction motors and asynchronous generators is well known. It is not proposed to describe his method in these pages, as this cannot be done in a few words; but it may be stated that, not only the *amount* of the armature current, but also the *power factor* is taken into account, and actual tests have shown that, even on zero power factor, the pressure at terminals will remain almost constant for any current up to the full-load current taken from the machine.†

Another method, due to Mr. Miles Walker, which however, is not applicable to a machine on a load of low power factor, consists in utilising the armature reaction to *strengthen* instead of weaken the field. The principle is very simple, and, if the action illustrated by Fig. 34 has been properly understood, the diagram Fig. 35 should serve to explain Mr. Walker's method.

It will be noticed that the field coils do not entirely encircle the polar projections, but are wound upon the shaded part only, leaving a certain portion without any winding upon it. If we suppose the heavy rectangle to represent the armature coil at the moment when the current is at its maximum,* it will be seen that the armature current tends to *weaken* the N pole and *strengthen* the S pole. But the portion of the N pole within the coil is nearly saturated by the magnetism due to the field coils,

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* See art. 12, Chap. II., p. 39.

† The reader who cares to go further into this matter is referred to Mr. Eborall's articles in the *Electrician* of July 3 and 10, 1903, and July 15, 1904; also to the excellent paper on compensated alternate-current generators read by Mr. Miles Walker before the Manchester Section of the Institution of Electrical Engineers, Nov. 29, 1904.

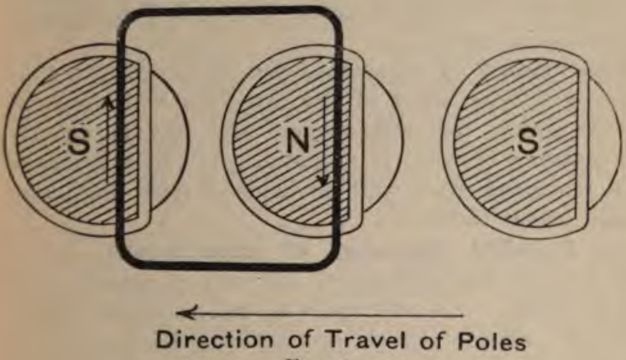

FIG. 35.

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* This rectangle should be considered as indicating by its position the *resultant* effect of the armature ampere turns. It has been drawn to indicate a power factor below unity: if the load were supposed non-inductive, the rectangle should be drawn with its sides through the centres of the poles. It is not easy to state clearly what is to be considered as the resultant ampere turns of a three-phase generator; but for the purpose of dealing with armature reaction, it is necessary to know what these amount to. It has been determined experimentally that if S stands for the number of conductors per pole and per phase, and if C is the $\sqrt{\text{mean square}}$ value of the current in any one phase, then the armature ampere turns may be taken as

$$\sqrt{3} \times \sqrt{2}\ S\ C,$$

or,

$$2{\cdot}45\ S\ C.$$

whereas the portion of the S pole under the armature coil is unsaturated (being, in fact, not magnetised by the field coils). The result is that the armature current will only slightly diminish the resultant number of lines issuing from the shaded portion of the N pole, but will induce a strong field in the unsaturated portion of the S pole, and so maintain, or even increase, the total flux through the armature coil.

This method is ingenious, but it is evident that the magnetising effect of the armature will *diminish* as the power factor decreases, and this is exactly the contrary of what is required. At the same time such machines work well on loads of fairly high and steady power factors.

(34) **Parallel Running of Alternators**.—It is well known that alternators will run in parallel, but not in series. An attempt will be made to briefly state the reasons which account for the successful parallel running of machines of the type we have been considering; but it is not intended to investigate in detail the actions that take place under various conditions of driving or of load.

In the first place, it must be recognised that the synchronous alternator is a reversible machine—that is to say, it will run as a motor when supplied with power from an alternating source. The reasons which account for the continuous-current dynamo being a reversible machine should be well known to every reader of these pages, and it should, therefore, be merely necessary to point out that when an alternator is used as a motor, exactly the same forces come into play between armature conductors and field, *provided the armature is supplied with* *current at the correct frequency, and that the revolving portion of the machine has been run up to the correct speed, known as the speed of synchronism.*

Imagine an alternator running as a generator on load at a definite speed, or frequency. The torque exerted by the prime mover is—as in the case of a dynamo—required to force the conductors carrying current across the magnetic field.* If, now, we suppose the prime mover to be disconnected, and, before the generator has had time to slow down, we connect the armature terminals to an alternating-current supply, the machine will continue to run as a motor, provided the frequency of the supply is correct—*i.e.*, such as to correspond with the speed and the number of poles.

The current will flow in the armature coils in the reverse direction, for corresponding positions of armature coils and poles; the result being a number of impulses tending to keep the machine *in step*. If load is thrown on the machine, there will be a momentary retardation, causing a slight displacement of phase between the back E.M.F. of the armature and the applied potential difference: these forces will not be exactly opposed to each other, and the result will be an increase of current which will be sufficient to keep the machines in step. If the machine breaks out of step, which might occur, for instance, on a sudden and considerable overload, then, since its speed is no longer that of synchronism, the regular succession of impulses is replaced by a number of irregular impulses in both directions, and the machine will quickly come to rest.

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* It is true that the forces exerted in the case of a single-phase alternator are intermittent, being of the nature of a series of impulses, and not continuous as in a dynamo, or—to a great extent—in a polyphase machine; but this does not alter the argument.

Although the synchronous alternating-current motor is an excellent machine for certain purposes, it has the disadvantage of not being self-starting. When running, it has one definite speed, depending upon the frequency of the supply circuit.

With regard to the parallel running of synchronous generators, as far as general principles are concerned, there is little to add to the foregoing remarks. If two or more machines are electrically coupled to common 'bus bars, and if any one of them—owing to a momentary slowing of its prime mover, or from any other cause—lags, only for an instant, behind the others, there will be an immediate rush of current from the 'bus bars, which will be sufficient to pull it into step. Successful parallel running depends less upon the design of the alternators than of the engines; an even turning moment is essential. Large and small units can be run in parallel with the greatest ease; and, although it is desirable that all the machines should give approximately the same shape of E.M.F. wave, this is not essential. The portion of the total load taken up by each of several generators in parallel will depend simply upon the amount of power developed by the respective prime movers. There is also a correct value of the exciting current for each of the several units corresponding to a given load; otherwise the necessary magnetic flux will have to be produced by internal circulating armature currents provided by the other machines. It is true that, before switching an additional machine on to the 'bus bars, it has to be run up to the speed of synchronism; but, once switched in, it is magnetically locked with the other machines, and to those who have had no experience of alternators running in parallel, it is surprising to note how firmly each unit appears to be gripped by the other machines and held in step.

The reasons which make it impossible to run synchronous alternators when coupled in series do not require a detailed explanation: the current being necessarily the same through all the armature windings, no interchange of synchronising currents can take place between the various machines, and the latter are, therefore, not magnetically locked together as would be the case if they were coupled in parallel.

(35) **Output of Polyphase Generators.** — A generator wound to give two or more currents differing in phase, will have a larger output than if it is wound to give only single-phase currents. It is not proposed to study and compare the various methods of winding armatures, in order to show that the polyphase generator has a greater output than the single-phase machine, but it is found in practice that, for the same type and dimensions, a machine provided with windings for two-phase currents will have an output about 30 per cent. greater than if wound for single-phase currents.

If single-phase currents are required, they can always be obtained from polyphase machines. Thus, a two-phase generator would work as a single-phase machine with its two windings coupled in series, the terminal pressure being $\sqrt{2}$ times the pressure per phase, for the same speed and excitation. The output, however, would not be so great, because, although the current, for the same heating effects, would be the same as before, the terminal pressure is not doubled by connecting in series the two (out of phase) windings.

The power of the two-phase machine = 2 (E × C), where E and C stand respectively for the pressure and current per phase—the load being assumed non-inductive. The power of the same machine used as a single-phase alternator will only be $\sqrt{2}$ E × C for the same losses in the armature coils, or about 30 per cent. less.

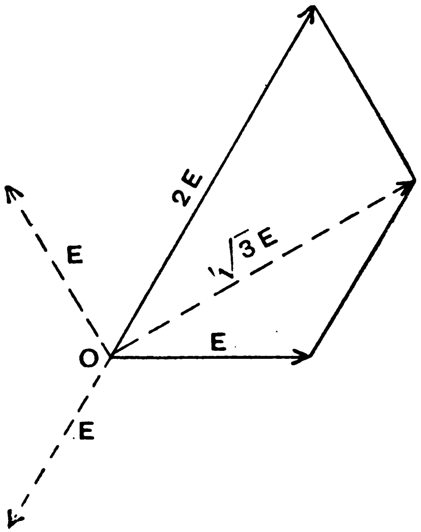

FIG. 36.

In the case of a three-phase generator, we may connect all three windings in series in the manner indicated by the vector diagram Fig. 36; and, if the

E.M.F. in each section is equal to E volts, the resultant E.M.F. will be 2 E. The output, for a current C in the armature conductors, will be $2 \text{ E} \times \text{C}$ instead of $3 (\text{E} \times \text{C})$, which would be the output for the same heat dissipation in armature coils if the machine were connected up for three-phase working.

A slightly greater output as a single-phase machine will be obtained by utilising only two of the three sections. The resultant E.M.F. will only be $\sqrt{3}$ E, as shown by the dotted vector in Fig. 36; but for equal $\text{C}^2 \text{R}$ losses in armature windings the current may now be

$$\text{C} \times \frac{\sqrt{3}}{\sqrt{2}},$$

making *output*

$$= \frac{\sqrt{3} \text{ E} \times \sqrt{3} \text{ C}}{\sqrt{2}}$$

$$= \frac{3}{\sqrt{2}} \text{ E C}$$

$$= 2{\cdot}13 \text{ E C}.$$

(36) **Double-Current Generators** — It is sometimes convenient and economical to use a single machine to generate both continuous and alternating currents, the latter being either single-phase or polyphase, as the case may require.

Such machines are practically identical with rotary converters, to which reference will be made in a later chapter, the only difference being that suitable means of mechanically driving the machine must be provided.

The double-current machine has the objection that it is difficult, if not impossible, to design it so as to be both a perfect dynamo and a perfect alternator.

Also, the speed, or number of poles, or both, must be great unless the required frequency is low; and the alternating-current pressure must always bear a definite relation to the direct-current pressure. If it is desired to regulate the alternating and continuous current circuits independently, it is necessary to insert suitable regulating devices in the connections after these have left the terminals of the machine.

The chief difference, electrically, between the rotary converter and the double-current generator is that, in the former, there is practically no armature reaction producing field distortion whereas, in the latter, both currents act together to produce field distortion; and it is partly for this reason that separate field excitation is frequently advisable.

The $C^2 R$ losses in the armature windings can be calculated for any given relation between the amounts of the alternating and continuous currents taken from the machine. If C_a is the maximum value of an alternating current which varies in a simple periodic manner, the $\sqrt{\text{mean square}}$ value of the current is $\frac{C_a}{\sqrt{2}}$. If C_d is a constant and unidirectional current, its $\sqrt{\text{mean square}}$ value is the same as its maximum value—*i.e.*, C_d; but if a wire carries an alternating current superimposed on a continuous current, the $\sqrt{\text{mean square}}$ value—which determines the heating losses—will depend upon the relative amounts of the two currents.

It can be shown that the $C^2 R$ losses in the armature of a double-current generator depend, not only upon the amount of the alternating or polyphase current relatively to the total (apparent) output of the machine, but also upon the *phase* of this current—*i.e.*, upon the *power factor* of the alternating or polyphase load.

Take as an example a double-current machine giving three-phase currents from three slip rings, and continuous currents at the commutator. If the direct-current voltage is taken as 100, the alternating-current pressure between phases will be 61·23 volts.*

Let us suppose the total current output to be 100 amperes, and the armature resistance ·01 ohms; then, if the machine is loaded to its full output on the direct-current side only, the armature $C^2 R$ losses will be $(100)^2 \times \cdot 01 = 100$ watts.

Now suppose we take the full output, of ten apparent kilowatts, from the three-phase slip rings. The amperes in each of the three armature sections will be

$$\frac{10{,}000}{3 \times 61\cdot 23} = 54\cdot 5 \text{ amperes,}$$

and the resistance of each section will be $\cdot 01 \times \frac{4}{3}$, which gives us for the total armature losses in the three sections

$$3 \times (54\cdot 5)^2 \times \cdot 01 \times \frac{4}{3} = 118\cdot 75 \text{ watts,}$$

which is greater than the heating loss when the whole of the output is on the direct-current side.

We are evidently not—in the above example—concerned with the *power factor* of the three-phase load, since it is the amount of the *current* only which determines the heating of the armature conductors; but when the load is taken from the commutator at the same time as from the slip rings, the power factor has to be taken into account in determining the total armature losses.*

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* See later chapter dealing with rotary converters for explanation of pressure ratios in these machines.

It will be found that, for the same total *apparent* output, and a given relation between alternating and direct current output, the armature losses will always be greatest when the power factor is unity (cos $\theta = 1$), and they will diminish with lower power factors.

In this connection the following figures may prove interesting; they are calculated for a three-phase double-current machine, on the assumption of the simple periodic wave form.

The total *apparent* output is assumed constant, the particulars of the machine being as assumed above; but instead of the whole load being either three-phase or continuous, we shall suppose the apparent output on the three-phase side to be equal to the direct-current output; the watts lost in armature conductors will then be approximately:

95 for cos $\theta = 1$

87·5 for cos $\theta = {\cdot}9$

and only 25 for cos $\theta = 0$.

For other ratios of the three-phase to the direct-current load the figures would be different.

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* See the writer's article, explaining this point by means of diagrams in the *Electrical World* of New York, Aug. 12, 1905, p. 265.

CHAPTER IV.

MEASUREMENT AND CALCULATION OF POWER ON POLYPHASE CIRCUITS.

(37) As a clear understanding of this portion of our subject is important, a short chapter will be devoted to the consideration of power measurement on two and three phase circuits.

It is not proposed to describe in detail the principles of construction of the various types of wattmeters used for measuring power on alternating-current circuits. The single-phase wattmeter may be considered as a modified form of Siemens dynamometer, with fixed thick-wire coil and movable fine-wire coil. The fixed coil is traversed by the main current, while the movable coil is connected in series with a large non-inductive resistance across the terminals, and takes a current proportional to the potential difference. The object of the non-inductive resistance is to reduce the "time constant"* of the

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* It was explained in article 12 how the *time* required for the growth or decrease of the current depends upon the self-induction of the circuit; but the effects of self-induction will depend very much on the *resistance* of the circuit. If the resistance is only high enough, the back E.M.F. of self-induction will become a negligible quantity in comparison with the E.M.F. required to overcome the resistance, and the *time-constant* of a circuit may be defined as the ratio of its coefficient of self-induction to its resistance, or

$$\textit{time constant} = \frac{\mathrm{L}}{\mathrm{R}}.$$

It follows that, in a circuit of which the time constant is small, the effects of self-induction may be negligible whatever may be the actual value of the E.M.F. induced by the passage of unit current.

shunt circuit to a negligible amount, in order that the current in this circuit may always be in phase with the potential difference across the terminals.

If C is the current in the fixed coil, and A the current in the movable coil, the reading of the wattmeter will be proportional to A C cos θ, where θ is the phase angle between the two currents; and, since A is strictly proportional to the terminal potential difference, the instrument will indicate the true power of a single-phase alternating-current circuit.

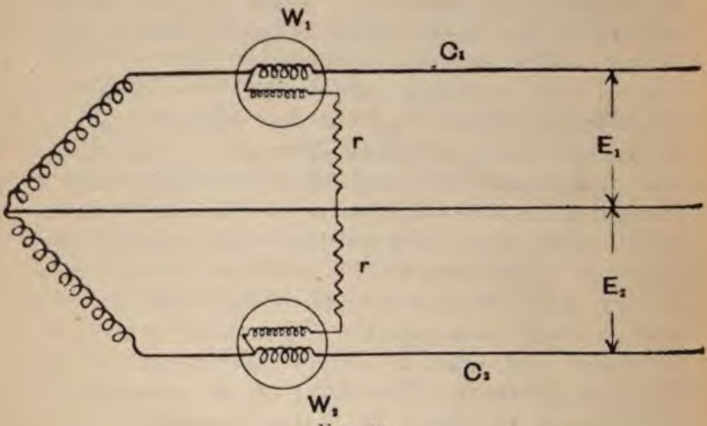

FIG. 37.

(38) **Power of Two-Phase Circuit.**—If the two circuits are separate—*i.e.*, if the distribution is by means of four wires—two wattmeters would be required—one in each circuit—and the sum of their readings would give the total power of the two phases.

If the load is balanced, as will be the case if it consists of induction motors only, the pressure, current, and angle of lag in the two phases will be the same, and one wattmeter will suffice; it is merely necessary to double its reading to obtain the total power of the two-phase circuit.

If there is a common return conductor for the two phases, one wattmeter will also suffice, provided the load is balanced. If the distribution of load is unequal, two wattmeters are required, as shown in Fig. 37.

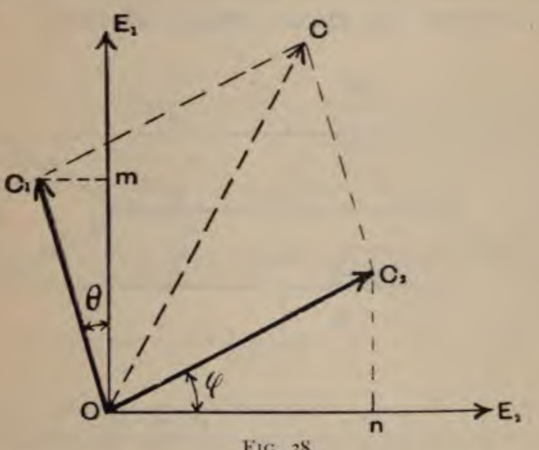

FIG. 38.

The vector diagram for a partly inductive unbalanced load has been drawn in Fig. 38.

Here $\overline{O\,E_1}$ and $\overline{O\,E_2}$, drawn at right angles to each other, and equal in length, represent the pressures E_1 and E_2, with a phase difference of 90 deg.

will be equal, and they will also be equally displaced in phase relatively to the corresponding E.M.F.'s.

There are two cases to be considered: (1) the case of star-connected generators with neutral point available, and (2) the case of mesh-connected machines in which there is no neutral point available. The arrangement shown in Fig. 40 is applicable to case (1), if we imagine the lamp load between the various phases and the common conductor, D, to be done away with. The load being balanced, the currents in A, B, and C will be equal, and equally displaced in phase relatively to the respective E.M.F.'s measured between each of the three terminals and the neutral point: the readings of the three wattmeters, W_1, W_2, and W_3, will be identical, and any one of these meters will, therefore, suffice to measure the load on a balanced three-phase circuit; it will be merely necessary to multiply the reading by three.

Consider, now, case (2), in which the neutral point is not available. The arrangement is as shown in Fig. 39, with the exception that one of the wattmeters — say W_2 — is supposed to be entirely dispensed with. The remaining wattmeter, W_1, with the free end of the fine-wire coil connected to B (as shown in the figure), will indicate the quantity E C cos (30 deg. $-$ θ), where θ is the angle of lag, (cos θ being the power factor). If, now, we remove the free end of the fine-wire coil from B to C—leaving the series coil in circuit with the conductor A—the instrument will indicate the quantity E C cos (30 deg. $+$ θ). The sum of these two expressions—after the necessary simplifications have been effected—amounts to $\sqrt{3}$ C E cos θ, which is

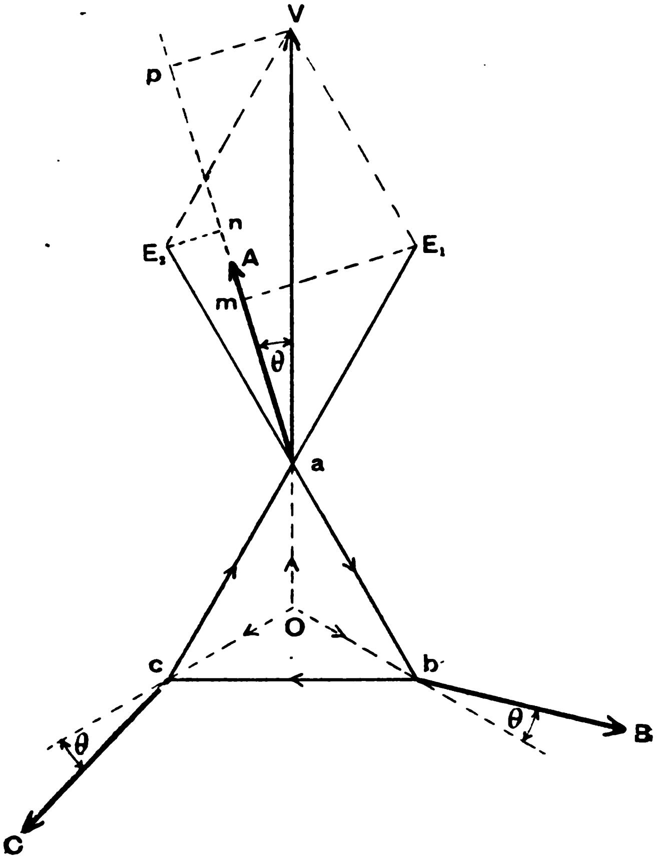

FIG. 41.

the expression for the total power, where C and E stand respectively for the current in any one conductor and the pressure between phases of a balanced three-phase system. Thus, even when the neutral point is not available, the sum of two readings taken on a single wattmeter will give us the true power of the balanced three-phase circuit.

The vector diagram Fig. 41 will make this clear to those readers who do not care to follow out the trigonometrical proof. Here $a\ b\ c$ is the E.M.F. or pressure triangle, and $\overline{a\,A}$, $\overline{b\,B}$, and $\overline{c\,C}$ are the three (equal) current vectors, the angle of lag in each case being θ.*

The power, as measured by the wattmeter with the free end of the shunt coil connected to the conductor B, will be $(a\ A) \times$ the projection of $a\ b$ (or of $a\ E_2$) on $a\ A$—*i.e.*, $w = (a\ A) \times (a\ n)$.

When the free end of the shunt coil is connected to the conductor C, the wattmeter will indicate

$$w = (a\ A) \times (a\ m).$$

The sum of these two quantities gives us

$$W = (a\ A) \times (a\ n + a\ m).$$

---

* Should the reader experience any difficulty in understanding why θ is the phase angle between current and E.M.F., he must consider the E.M.F. triangle $\overline{a\ b\ c}$ to be replaced by the equivalent *star* system of vectors $\overline{O\,a}$, $\overline{O\,b}$, $\overline{O\,c}$. If the load is non-inductive, the current vectors will be in the same direction as their respective E.M.F. vectors; but on an inductive load they will make an angle θ with the E.M.F. vectors such that $\cos\theta =$ the power factor, or the ratio of real to apparent output.

It should be clearly understood that the sides of the triangle $a\ b\ c$ represent, by their length and direction, the magnitude and phase relations of the pressures *measured between conductors*, while the star $O\ a$, $O\ b$, $O\ c$ represents the equivalent system of vectors all radiating from a common neutral point, O. That such a system of vectors—radiating from a common point, O, and terminating at the points a, b, and c respectively—is equivalent to the triangle of vectors, is proved by the fact that $\overline{a\ b}$ is (vectorially) equal to $(\overline{O\ b} - \overline{O\ a})$. Similarly,

$$\overline{b\ c} = \overline{O\ c} - \overline{O\ b}$$

and

$$\overline{c\ a} = \overline{O\ a} - \overline{O\ c}.$$

But the latter quantity is evidently equal to O p, or to the projection of the *resultant E.M.F. vector* $\overline{(O\ V)}$ upon a A; hence

$$W = (a\ A) \times (a\ V) \cos \theta,$$

and if E stands for the magnitude of the pressure between phases—represented in the diagram by the sides of the triangle $\overline{a\ b\ c}$, or the length of the vectors $\overline{a\ E_1}$ or $\overline{a\ E_2}$—then it can be easily shown that $a\,V = \sqrt{3}\,E$, which gives us for the sum of the two wattmeter readings

$$W = \sqrt{3}\,E\ (a\ A) \cos \theta.$$

This is the expression for the total power of the three-phase balanced circuit, being equal (as may be verified by an inspection of Fig. 41) to 3 (O a) × (a A) cos θ.

The method of taking two readings on a single wattmeter would not be convenient in practice, neither is it necessary. It is possible to combine the shunt currents due respectively to the E.M.F.'s E_1 and E_2 (Fig. 41) so as to obtain a total current in the fine-wire coil proportional in amount and similar in phase to the E.M.F. represented by the resultant vector $\overline{a\,V}$.

Instead of changing over the shunt connection of the single wattmeter from B to C (Fig. 39), it is merely necessary to provide two non-inductive resistances *each* of r ohms (as before), and permanently connect them respectively to the two conductors in which there is no wattmeter, as shown in Fig. 42. The wattmeter will then directly indicate the total power of the balanced circuit.

This arrangement is very similar to an alternative method which consists in producing an *artificial neutral point* by means of a star resistance, or equivalent arrangement in which a choking coil is used: it is, indeed, only necessary to imagine a third resistance, of *r* ohms, inserted between the fine-wire coil and the point O. This—on the assumption that the shunt-coil resistance is negligible—would have the effect of reducing the current in the shunt coil to one-third of its previous value, and the wattmeter would, therefore, indicate one-third of the total power, exactly as if it were connected to the neutral point of a star-connected generator supplying a balanced load.

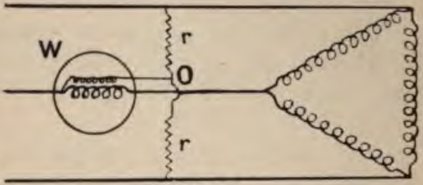

FIG. 42.

(40) **Vector Diagram for Calculating Star Resistances**.—The diagram Fig. 43 will serve to indicate how the required non-inductive resistance may be calculated to give any desired shunt current through the wattmeter or other instrument connected in one phase only.

Let *a b c* be the E.M.F. triangle—the sides of which are equal, and indicate the pressures between phases—while *c* C represents the required current in shunt coil.

The first and all important point to bear in mind in the construction of such diagrams is that the current, C, leaving any one terminal must exactly equal the resultant of the other two currents, A and B; in other words, the sum of the currents at the three terminals is always equal to zero.* The first condition to be fulfilled in Fig. 43 is, therefore, that the three current vectors *a* A, *b* B, and *c* C shall form a closed triangle.

Draw the dotted line B A parallel to the side *b a* of the pressure triangle, and at a distance below it equal to half the length of the (known) current vector *c* C. If, now, we join any point such as O_1—lying on the prolongation of the vector *c* C, within the triangle *a b c*—to the vertices *a* and *b* of the pressure triangle, it will be seen that the prolonga-

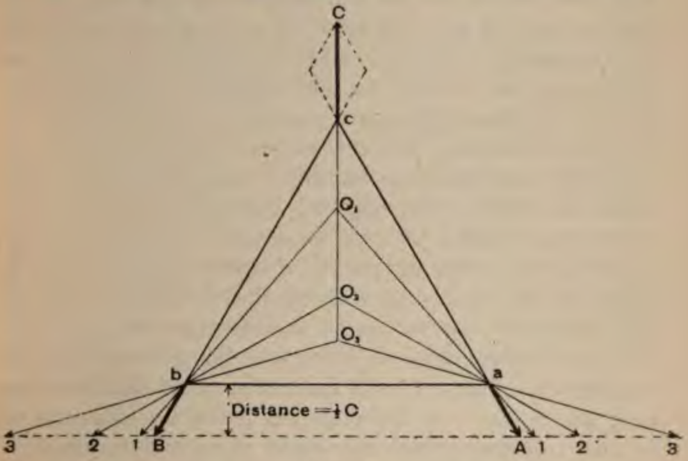

FIG. 43.

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* The only exception being the case of a star-connected generator with a fourth conductor returning to the neutral point.

tion of the lines until they cross the dotted line B A will give us the current vectors *a* 1 and *b* 1, which, when combined together, will exactly balance *c* C.

It will be seen that there are many solutions of the problem, but the required ohms in the three arms forming the star resistance are readily calculated for any position of the point O.

The drop of pressure in each arm of the non-inductive resistance is represented by the lengths of the vectors O *a*, O *b*, and O *c* (in phase with the corresponding currents), and the necessary ohms in each arm can readily be calculated by dividing the lengths of these pressure vectors by the lengths of the corresponding current vectors—the scale to be used for the pressure vectors being, of course, the same as used for drawing the triangle *a b c*.

The total watts lost in the star resistance will be equal to the sum of the $C^2 r$ losses in the various arms of the star. It will be found that, if *c* C is kept constant, these losses increase as the point O is taken lower down. They will be greater for the system of vectors drawn from the point O_3 than for that which has for the common point O_2 (the centre of the triangle); but the most economical arrangement consists in doing away with the resistance in series with the current C. This brings the point O to coincide with *c*, and makes the total power absorbed by the resistances only two-thirds of what it would be if a star resistance with three equal arms were used. It will be observed that the connections in this case would be exactly as shown in Fig. 42.

Example.—Let us assume the pressure between phases to be 100 volts. This means that the triangle *a b c* is drawn to such a scale that each side measures 100 units in length. Let us also suppose the required current, *c* C, to be ·2 amperes.

From the point O_2—which is the centre of the triangle — draw the three lines radiating to the corners. The three pressure vectors, $\overline{O_2\,a}$, $\overline{O_2\,b}$, $\overline{O_2\,c}$, will all be equal to 57·7, and the three current vectors, $\overline{c\,C}$, $\overline{a\,2}$, $\overline{b\,2}$, will all be equal to ·2.

The three resistances forming the star will each have to be equal to $\frac{57{\cdot}7}{{\cdot}2} = 288{\cdot}5$ ohms; and the total watts dissipated will be

$$W = 3 \times 57{\cdot}7 \times {\cdot}2 = 34{\cdot}62 \text{ watts.}$$

Now reduce the resistance in series with the current C until it is equal to zero: this will bring the point O to coincide with *c*, and the two pressure vectors representing the drop across the two remaining arms of the star resistance, will be $\overline{c\,a}$ and $\overline{c\,b}$—each equal to 100 volts. The currents $\overline{a\,A}$ and $\overline{b\,B}$, in phase with these pressures—and such that they will balance the current $\overline{c\,C}$—will each be equal to ·1154 amperes.

This makes each of the two resistances equal to $\frac{100}{{\cdot}1154} = 866{\cdot}55$ ohms; and the total watts dissipated will be

$$W = 2 \times 100 \times {\cdot}1154 = 23{\cdot}08 \text{ watts,}$$

which is exactly two-thirds of the power lost in the resistance with three equal arms.

(41) **Vector Diagram for Unbalanced Non-Inductive Three-Phase Load.**—Imagine a non-inductive load, such as a bank of incandescent

lamps, to be connected, delta fashion, between the terminals of a three-phase supply.

Let m, n, and p stand respectively for the values of the currents flowing through each batch of lamps. The vectors representing the currents (see Fig. 44)

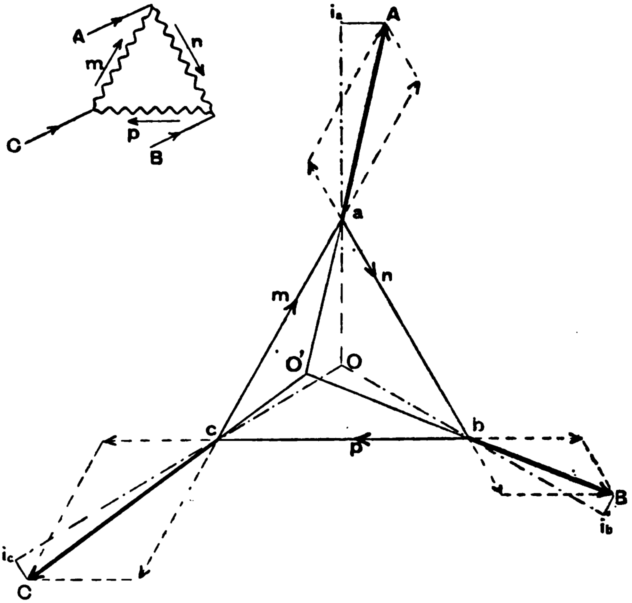

FIG. 44.

will be $\overline{c\,m}$, $\overline{a\,n}$, and $\overline{b\,p}$, drawn in phase with the pressures between terminals, and of such lengths as to represent, to a suitable scale, the magnitude of the respective currents.

The total currents which will flow in the three conductors connecting the load to the source of supply will be A, B, and C, which are readily calculated by suitably combining the currents m, n, and p. Thus,

$$A = m - n$$

$$B = n - p$$

$$C = p - m$$

And by constructing the dotted parallelograms in Fig. 44 we readily obtain the three vectors $\overline{aA}$, bB, and $\overline{cC}$, which, by their lengths and directions, indicate the magnitude and phase relations of the currents in the three conductors of the transmission line.

Owing to the construction of the diagram, these three vectors will necessarily always form a closed triangle, which is one of the fundamental conditions to be fulfilled, because this is equivalent to stating that the total of the currents leaving the source of supply by any two of the conductors must be equal to the current returning to the source of supply by the third conductor.

Another condition which will always be fulfilled if the load is non-inductive, is that the current vectors, when produced inside the triangle, will meet at a common point O′. It has already been explained that the E.M.F. triangle a b c can be replaced by an equivalent system of vectors radiating from a common point and terminating at the points a, b, and c respectively; and *if the load is non-inductive*, there must, of necessity, be such a system of vectors, *each one of which will be in phase with the correspondng current vector.* In the diagram, this particular system of pressure vectors is represented by the lines O′a, O′b, and O′c.

Calculation of Power from the Diagram.—A number of different sets of measurements can be taken off the diagram Fig. 44, by means of which the total power, W, can be calculated. In the first place, we have

$$W = (c\,a) \times (c\,m) + (a\,b) \times (a\,n) + (b\,c) \times (b\,p)$$

But, if the currents m, n, and p are not known, and we are only given the three current vectors A, B, and C, we have merely to produce these until they meet at the common point O′: then the total power is

$$W = (O'\,a) \times (a\,A) + (O'\,b) \times (b\,B) + (O'\,c) \times (c\,C).$$

It should, perhaps, be pointed out that *any* star system of pressure vectors may be used, provided they terminate at the points a, b, and c respectively; but if these vectors radiate from any other point than O′, the phase angles between each vector and the corresponding current vector must be taken into account; each pressure vector must be multiplied by the *projection* upon it of the corresponding current vector. We can, therefore, also write:

$$W = (O\,a) \times (a\,i_a) + (O\,b) \times (b\,i_b) + (O\,c) \times (c\,i_c),$$

and any one of the above three expressions will correctly give us the total three-phase power.

Wattless Current.—If the load were balanced—*i.e.*, if all three currents A, B, and C were equal—the vectors $\overline{a\,A}$, $\overline{b\,B}$, and $\overline{c\,C}$ (for the condition of a load which is non-inductive) would lie on the lines drawn from the centre O through the vertices of the triangle. The reader may, therefore, at first sight, be unable to account for the presence of what would appear to be "idle" or "wattless" current components in the diagram for a *non-inductive* load (see Fig. 44), even though this be unbalanced.

It will be noticed, however, that, in the diagram Fig. 44, the wattless component $\overline{i_a \text{A}}$ of the current A is *negative* (being measured to the right of the pressure vector $\overline{\text{O}\, a}$), while the other two wattless components, $\overline{i_b \text{B}}$ and $\overline{i_c \text{C}}$, are both *positive*; and it will also be found that the sum of these two positive components is exactly equal to $\overline{i_a \text{A}}$, thus making the mean idle current in the three phases equal to zero. This can be proved as follows:

The line i_a A is the projection of $c\, m$ on $c\, b$ *less* the projection of $a\, n$ on $c\, b$, and we may therefore write

$$i_a \text{A} = m \cos 60 \text{ deg.} - n \cos 60 \text{ deg}$$

Similarly,

$$i_b \text{B} = n \cos 60 \text{ deg.} - p \cos 60 \text{ deg.}$$

and $$i_c \text{C} = p \cos 60 \text{ deg.} - m \cos 60 \text{ deg.}$$

Summing up both sides of the three equations, we get

$$\begin{aligned} &i_a \text{A} + i_b \text{B} + i_c \text{C} \\ &\qquad = \cos 60 \text{ deg.} (m - n + n - p + p - m) \\ &\qquad = 0. \end{aligned}$$

But the simplest proof that the power factor is really unity evidently consists in producing the three current vectors A, B, and C until they meet at the point O′ within the triangle. If they do not meet at a common point, it is a certain indication that the load has appreciable induction, or capacity, or both.

(42) Diagram for Balanced Inductive Load.—

A condition very frequently met with in practice is

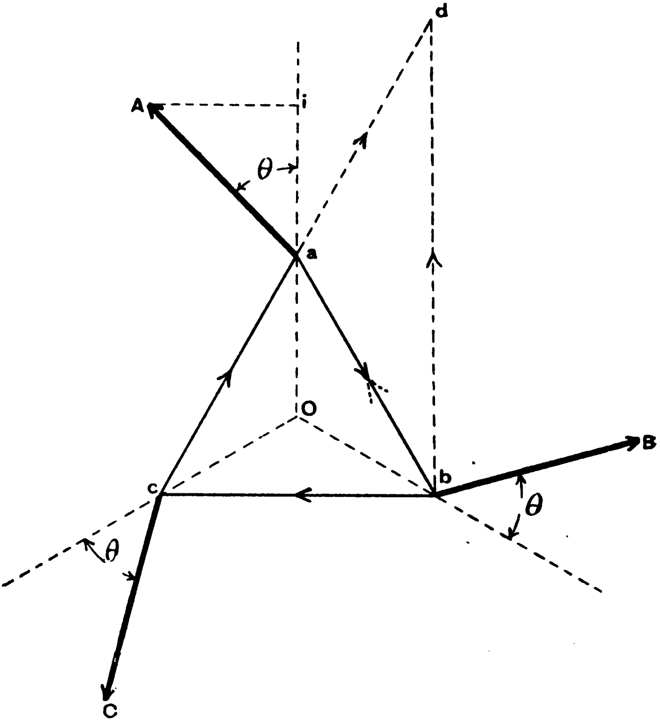

FIG. 45.

that of generators supplying current to induction motors only.

In this case the power factor will be less than unity; but the load will be equal on all three phases.

Such a load is shown diagrammatically in Fig. 45.

Here the current vectors A, B, and C are all equal in length; but, instead of being in phase with the pressure vectors $\overline{o\,a}$, $\overline{o\,b}$, and $\overline{o\,c}$, they lag behind these vectors by a certain angle θ.

The total power is

$$\begin{aligned} W &= 3\,(o\,a \times a\,i) \\ &= 3 \times o\,a \times a\,A \cos\theta \\ &= \sqrt{3}\,E\,C \cos\theta \end{aligned}$$

where E is the pressure between phases;

C is the amount of any one of the three line currents; and

$\cos\theta$ is the power factor of the balanced three-phase circuit.

Measurement of Power Factor on Balanced Load.—There are instruments called phase meters, or power factor indicators, so constructed as to show, by the position of a pointer, the phase angle θ between current and E.M.F.; but, by means of a voltmeter, ammeter, and wattmeter, this angle can evidently be calculated: the two former instruments will enable us to calculate the *apparent* power ($3 \times o\,a \times a\,A$), while the wattmeter will directly indicate the true power ($3 \times o\,a \times a\,i$), and the ratio which this latter quantity bears to the apparent power will be the *power factor* ($\cos\theta$) of the three-phase circuit.

It is, however, possible to determine the angle θ, even if neither ammeter nor voltmeter is available, by the use of a wattmeter only.

It has already been explained (see p. 118) how the total power of a balanced three-phase circuit may be measured by means of a single wattmeter connected in series with one main, and of which the shunt coil is connected alternately to each of the other two mains; the sum of the two readings giving the total power.

These two wattmeter readings will also enable us to calculate the power factor of the load. Let W_1 and W_2 be the two readings of the wattmeter, then, if θ is the angle of lag, it can be shown that

$$\tan \theta = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}.$$

From this we obtain the angle θ, and $\cos \theta$, the power factor of the balanced circuit.

That the above formula is correct for a power factor of unity is evident, because W_1 will be equal to W_2, thus making the numerator of the right-hand side of the equation equal to zero; and if $\tan \theta = 0$ it follows that $\theta = 0$, which is the condition required to make the power factor ($\cos \theta$) equal to unity. By the aid of Fig. 45 it will be seen that the above formula is correct for any value of the angle θ.

Let us suppose the current A to pass through the series coil of the wattmeter, and the reading W_1 to be taken with the free end of the shunt coil connected to c, while for the reading W_2 this connection is transferred to b.

The power W_1 will be equal to the product of the volts $c\ a$ by the current a A, or to the length $c\ a$ multiplied by the projection of a A upon $c\ a$. Similarly, the power W_2 will be equal to $a\ b$ multiplied by the projection of a A on $a\ b$.

It will be observed that, in the two readings, the *amounts* of the main and shunt currents remain the same, but the *phase* of the shunt current is different. The results obtained by adding or subtracting the two wattmeter readings would evidently correspond with a single reading of the wattmeter obtained by passing through the shunt coil a current equal to the sum or difference—as the case may require—of the two separate shunt currents. Now, the *sum* of the two vectors $\overline{c\,a}$ and $\overline{a\,b}$ is $\overline{c\,b}$; and their *difference*—which is the same thing as the sum of $\overline{c\,a}$ with $\overline{a\,b}$ *reversed*—is $\overline{b\,d}$, exactly at right angles to $b\,c$, and equal in length to $\sqrt{3}$ times $b\,c$. We may, therefore, write

$$
\begin{aligned}
(W_1 + W_2) &= b\,d \times \text{the projection of } a\,A \text{ upon } b\,d \\
&= b\,d \times a\,i \\
&= \sqrt{3}\,b\,c \times a\,i \quad . \quad . \quad . \quad . \quad . \quad (1)
\end{aligned}
$$

and $$(W_1 - W_2) = b\,c \times i\,A \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Inserting the values (1) and (2) in the original equation, we have

$$
\begin{aligned}
\tan\,\theta &= \frac{\sqrt{3} \times b\,c \times i\,A}{\sqrt{3} \times b\,c \times a\,i} \\
&= \frac{i\,A}{a\,i}
\end{aligned}
$$

which, since $a\,i\,A$ is a right-angled triangle, is the definition of the trigonometrical tangent of the angle θ, and proves the correctness of the formula.

(A little difficulty may be experienced in understanding why the *difference* of the vectors $\overline{c\,a}$ and $\overline{a\,b}$ should have been taken to obtain the equivalent resultant vector from which the *sum* of the wattmeter readings is calculated, and *vice versa*; but this is due to the method of changing over the shunt connection, leaving the *same* end of the shunt coil permanently connected to the main A in which the wattmeter is placed.)

Example. — Let us suppose that the wattmeter reading W_1 = 20 kilowatts and W_2 = 10 kilowatts, then

$$\tan\,\theta = \frac{\sqrt{3}\ (20-10)}{20+10}$$
$$= {}^{\cdot}577$$

Referring to a table of natural tangents, we see that this corresponds to an angle $\theta = 30$, and since cos 30 deg. = ·866, this will be the power factor of the balanced three-phase circuit.

(43) **Unbalanced Inductive Load** —Consider a three-phase system in which a lamp load is taken off one or more phases, in addition to induction motors on all three phases. This may be effected (1) by connecting the lamps in delta fashion directly between the conductors A B, B C, or C A, or (2) if the generator armature windings are star connected, by running a fourth wire, D, back from the load to the neutral point, and connecting the lamps between the terminals A, B, or C and this fourth conductor, as shown in Fig. 40 (p. 117). In either case the three current vectors A, B and C (Fig. 46) may be unequal in length, and make different phase angles, α, β, and γ, with the equivalent star vectors drawn from the point O (the centre of the triangle); but in case (1) these three vectors will form a closed triangle when combined together, whereas in case (2) the vector representing the current (if any) in the fourth conductor, D, will be required to close the polygon of the current vectors.

The question now arises as to what is to be understood by the power factor of such a load. It may still be defined as the ratio of the real power to the apparent power; but although the real power is readily measured or calculated as already explained, it is not so easy to define what is now to be understood by the apparent power of the unbalanced three-phase circuit.

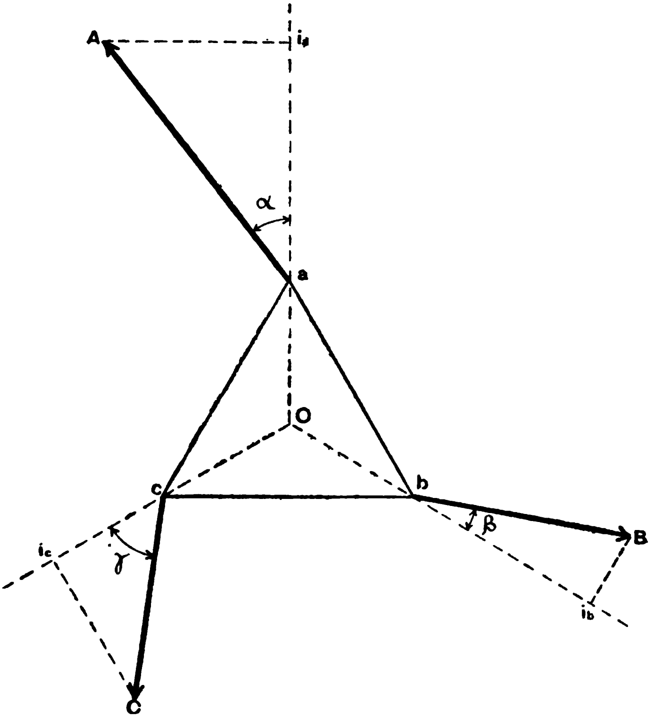

FIG. 46.

If all three phase angles, α, β, and γ, are measured in the direction of retardation, a close approximation to the power factor will be obtained by assuming the *apparent* output to be equal to

$$e \times (a\,\mathrm{A} + b\,\mathrm{B} + c\,\mathrm{C})$$

the *real* output being

$$\begin{aligned} \mathrm{W} &= e \times (a\,i_a + b\,i_b + c\,i_c) \\ &= e \times (\mathrm{A} \cos \alpha + \mathrm{B} \cos \beta + \mathrm{C} \cos \gamma). \end{aligned}$$

Hence power factor $= \dfrac{(\mathrm{A} \cos \alpha + \mathrm{B} \cos \beta + \mathrm{C} \cos \gamma)}{\mathrm{A} + \mathrm{B} + \mathrm{C}}$.

This expression is not, however, strictly correct; and this will be understood when it is realised that one or two of the phase angles may be measured in *advance* of the star vectors $\overline{o\,a}$, $\overline{o\,b}$, or $\overline{o\,c}$. This might occur if the load is nearly or quite non-inductive on one or more of the phases, and, indeed, the incorrectness of the formula will at once be apparent if we attempt to apply it to the case of an unbalanced but purely non-inductive load such as is represented by the vector diagram Fig. 44.

Here the power factor is unity, yet the sum of the three currents A, B, and C would be greater than the sum $\mathrm{A} \cos \alpha + \mathrm{B} \cos \beta + \mathrm{C} \cos \gamma$ (or $a\,i_a + b\,i_b + \mathrm{C}\,i_c$), which proves the above formula to be inaccurate, and not applicable to all cases of an unbalanced load.

It will be found, however, that if we consider the mean total current as being made up of two factors — the mean useful current and the mean "wattless" or "idle" current—we shall, by combining these at 90 deg., obtain an expression for the total apparent power which may be used for calculating the power factor in all cases, whether the load be balanced or otherwise.

Thus, the mean total line current, instead of being written $\frac{A + B + C}{3}$, should be expressed as

$$\tfrac{1}{3}\sqrt{(a + b + c)^2 + (i_a + i_b + i_c)^2}$$

where a, b, and c stand for the "work" components of the currents—$A \cos \alpha$, $B \cos \beta$, and $C \cos \gamma$—and i_a, i_b, and i_c stand for the "idle" components, at right angles to these last, or $A \sin \alpha$, $B \sin \beta$, and $C \sin \gamma$. When the *sign* of this idle current component is taken into account, it will be seen that the above expression may give results differing appreciably from the mean of the currents in the lines, and, in the event of the load being entirely non-inductive, the algebraic sum of the idle currents will be zero, thus making the apparent power, as calculated by this method, equal to the true power.

The power factor of an unbalanced three-phase load may, therefore, be expressed by the ratio

$$\frac{\text{real power}}{\sqrt{(\text{real power})^2 + (\text{``idle power''})^2}}$$

which, if we adopt the lettering of Fig. 46, would be written

$$\frac{(a\,i_a + b\,i_b + c\,i_c).}{\sqrt{(a\,i_a + b\,i_b + c\,i_c)^2 + (A\,i_a + B\,i_b + C\,i_c)^2}}$$

Example.—Suppose the pressure between terminals to be 500 volts, and let the phase angles be respectively

α = 45 deg. of *lag*
β = 10 deg. of *lag*
γ = 15 deg. of *lead*

and assume the main currents, as indicated by the line ammeters, to be

$$\begin{aligned} A &= 140 \text{ amperes.} \\ B &= 215 \quad ,, \\ C &= 105 \quad ,, \end{aligned}$$

Then

$$\begin{aligned} a\ i_a &= A \cos \alpha = 99 \\ b\ i_b &= B \cos \beta = 211{\cdot}7 \\ c\ i_c &= C \cos \gamma = 101{\cdot}4 \\ \hline \text{Total} & \ldots\ldots\ldots\ldots\ 412{\cdot}1 \end{aligned}$$

Also

$$\begin{aligned} A\ i_a &= A \sin \alpha = +\ 99 \\ B\ i_b &= B \sin \beta = +\ 37{\cdot}3 \\ C\ i_c &= C \sin \gamma = -\ 27{\cdot}2 \\ \hline \text{Total} & \ldots\ldots\ldots\ldots\ 109{\cdot}1 \end{aligned}$$

The real power is therefore

$$\frac{500}{\sqrt{3}} \times 412{\cdot}1 = 119 \text{ kilowatts,}$$

and the power factor is

$$\frac{412{\cdot}1}{\sqrt{(412{\cdot}1)^2 + (109{\cdot}1)^2}} = {\cdot}967.^*$$

---

* If it is desired to dispense with the process of squaring and extracting the square root, we can use trigonometrical tables thus:

Let $\cos \theta$ stand for the power factor of the three-phase circuit: then, since

$$\text{mean } \sin \theta = \tfrac{1}{3} \times 109{\cdot}1$$

and

$$\text{mean } \cos \theta = \tfrac{1}{3} \times 412{\cdot}1$$

$$\tan \theta = \frac{109{\cdot}1}{412{\cdot}1} = {\cdot}2647$$

which corresponds to an angle θ of $14^\circ\ 50'$. By referring to a table of natural cosines we see that $\cos 14^\circ\ 50' = {\cdot}967$, which is the same result as obtained by the more lengthy process of adding the squares and extracting the square root.

In the March number of *Power* (1906) the author describes a simple graphical method which requires only four measurements to be taken off the diagram, and yet gives all particulars for calculating the "real" and the "idle" power, and hence the power factor of any three-phase circuit whether balanced or not.

(44) **Measurement of Power on High-Tension Circuits.**—In all the foregoing diagrams, the instruments—whether wattmeters or ammeters—have been shown as being connected in series with the main leads. This method, however, only applies to circuits of pressures not exceeding, say, 500 volts. For higher pressures, transformers are generally used, in order that low-tension connections only may be brought to the terminals of the instruments.

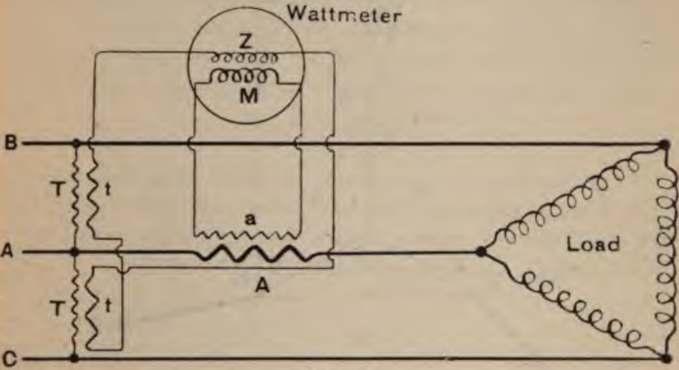

FIG. 47.

The various methods of power measurement are not altered thereby; it is merely necessary to consider the transformers as suitable pieces of apparatus for providing secondary pressures or currents—as the case may be—*exactly proportional, and of the same phase* as the primary pressures or currents.

As an example the diagram Fig. 47 has been drawn. It illustrates a suitable arrangement for measuring the total power of a *balanced* high-tension three-phase circuit.

Here A and *a* are the primary and secondary windings of the *current transformer* connected in series with one of the mains. This sends a current through the main coil, M, of the wattmeter, of the same phase as the current A, and of which the actual amount will depend upon the ratio of the primary and secondary turns, but which — in any case—will be proportional to A.

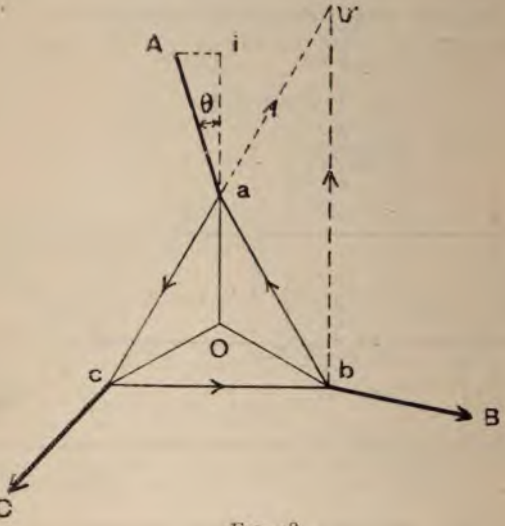

FIG. 48.

The two *pressure transformers*, each with T turns in the primary winding and *t* turns in the secondary, have their primaries connected up between A and B and A and C respectively, while the secondary windings are connected in series, and provide the shunt current for the pressure coil, Z, of the wattmeter.

Fig. 48 will serve to explain how the wattmeter can be made to correctly indicate the total output of the balanced circuit.

It will be seen that the secondary winding of one pressure transformer (Fig. 47) is *reversed* before connecting in series with the other transformer and the pressure coil of the wattmeter. This means that the resultant current through the pressure coil will not be proportional to the *sum* of the vectors $\overline{b\,a}$ and $\overline{a\,c}$ (Fig. 48), but to their *difference*, which is represented by the dotted vector $\overline{b\,v}$. This last is at right angles to $c\,b$, and parallel to $O\,a$; and, since the total output is equal to three times $O\,a \times A \cos\theta$, or to

$$3\,(O\,a) \times (a\,i),$$

it is evidently only a question of calibration and of the ratios of turns in the various transformer windings, to make the single wattmeter register the exact total output of the three-phase high-tension balanced circuit.

CHAPTER V

POLYPHASE TRANSFORMERS.

(45) **Theory of the Single-Phase Transformer.** A single-phase alternating-current transformer may be considered as consisting of a core of laminated iron upon which are wound two sets of coils, known as the primary and secondary windings respectively.

If an alternating E.M.F. is applied to the terminals of the primary, this will lead to a certain flux of alternating magnetism being set up in the iron core, which, in its turn, will induce a back E.M.F. in the primary winding; the action being that of a *choking coil* (see article 17, Chapter II.). But since the secondary circuit — although not in electrical connection with the primary—is wound on the same iron core as the latter, the variations of magnetic flux which induce the back E.M.F. in the primary, will, at the same time, generate an E.M.F. in the secondary coils.

The path of the magnetic lines is usually through a closed iron circuit; and, although in practice there is always a certain amount of leakage or stray magnetism which is not enclosed by the secondary windings, the effects of this magnetic leakage are very small in all well-designed transformers, and it will somewhat simplify the diagrams if we neglect this entirely. The assumption is, therefore, that the whole of the magnetism required to produce the necessary back E.M.F. in the primary coil passes also through the secondary coils—that is to say, the induced E.M.F. *per turn of wire* is supposed to be exactly the same in the secondary as in the primary coil.

Suppose now that the two ends of the primary winding are connected to constant-pressure mains, and that no current is taken from the secondary winding. Under these conditions, the primary circuit acts simply as a choking coil, of which the self-induction is so great, and the ohmic resistance relatively so small, that no current passes, except the very small amount required to magnetise the core. The induced E.M.F. is, therefore, practically equal and opposite to the applied potential difference at primary terminals, and the relation between the magnetic flux in the core and the primary impressed E.M.F. will be given by the equation

$$e_m = \frac{4 \mathbf{N} \mathrm{S} n}{10^8},$$

where e_m stands for the *mean* value, in volts, of the primary E.M.F. and n is the frequency.

The number of turns, S, in the primary of a well-designed transformer is always such that the current required to produce the magnetisation **N** is very small; it is generally somewhere between 2 per cent. and 5 per cent. of the full-load current.

Although the rise and fall of the magnetism will be a quarter of a period out of phase with the E.M.F., the open-circuit primary current will not be entirely "wattless," but may be considered as made up of two components—the "wattless" or true magnetising component, in phase with the magnetism, and the "work" component, due to hysteresis and eddy currents, in phase with the impressed E.M.F. The reader is, however, referred to article 23 (p. 59), where the question of magnetising current for a circuit containing iron has already been dealt with.

Since the secondary and primary coils are both wound on the same core, it follows that any variation in the magnetism will produce an E.M.F. in *both* circuits; also, the actual volts induced will be directly proportional to the number of turns of wire in the coil.

Thus, if the primary winding consists of 1,000 turns of wire, all in series, while the secondary has only 50 turns, the ratio of turns is 20 : 1, and this will also be the ratio of primary impressed E.M.F. to secondary induced E.M.F. on the assumption of there being no magnetic leakage.

For convenience, we shall, in all cases, suppose the primary and secondary windings to have the same number of turns; the transforming ratio will therefore be 1 : 1, and the pressure obtained at secondary terminals will be exactly equal, but opposite in phase, to the applied primary pressure.

This last statement will only be strictly correct if we also assume the ohmic resistance of the coils to be negligible: but the pressure drop due to this internal resistance is always small and rarely exceeds 2 per cent., even in small transformers; it is easily taken into account, if desired, but since the principles of action only will receive our attention, details of design will not be dealt with, and we may, therefore, neglect the ohmic resistance of both primary and secondary coils. Imagine, now, that the secondary circuit of a transformer, with primary on constant-pressure mains, is closed through a resistance; the resulting current will produce a magnetising force in the core. This magnetising force will not produce a change in the magnetism, because it will be instantly counteracted by a change in the primary current, which will so adjust itself as to maintain the same (or nearly the same) cycle of magnetisation as before; that is to say, the flux will continue to be such as will induce an E.M.F. in the primary windings equal but opposite to the primary impressed potential difference. These two opposing pressures can evidently not be *exactly* equal, or no current would flow in the primary coils; but since, in practice, the primary resistance — although not of zero value—is relatively small, it will be understood that a very small *resultant* pressure across the primary terminals would cause a very large current to flow through the coils.

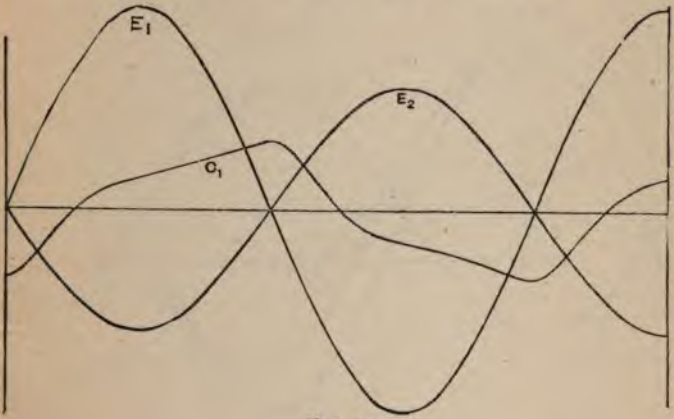

FIG. 49.

In Fig. 49, E_1 is the curve of primary impressed E.M.F., and C_1 is the magnetising current, distorted by the hysteresis of the iron core. E_2 is the curve of secondary E.M.F., which coincides in phase with the primary induced E.M.F., and is therefore—on account of the comparatively small ohmic resistance of the primary—almost exactly in opposition to the impressed E.M.F. The curve of magnetisation (not shown) would be exactly a quarter period in advance of the secondary E.M.F.

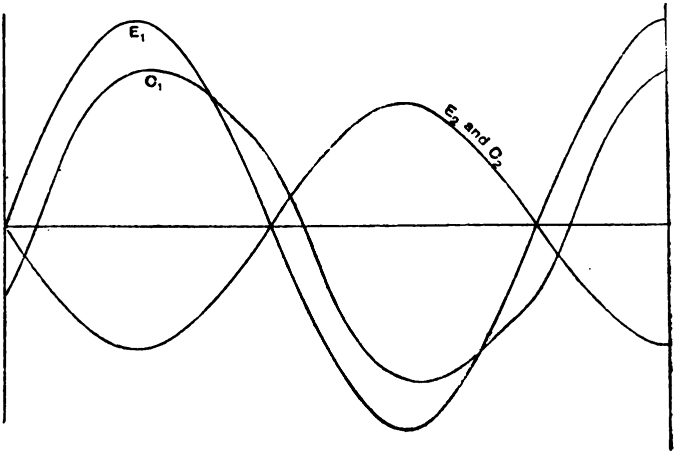

FIG. 50.

In Fig. 50 the secondary circuit is closed through its proper load of incandescent lamps. There is no appreciable self-induction in such a circuit, and the secondary current will, therefore, be in step with the secondary E.M.F. For simplicity it is represented in Fig. 50 by the same curve as E_2.

The tendency of this secondary current being to weaken the magnetism in the core, and therefore diminish the primary induced E.M.F., it follows that the current in the primary will grow until the magnetism is again of such an amount as to restore balance in the primary circuit. Hence, neglecting the small loss of volts due to the increased primary current, the induction through the primary must remain as before; and the new current curve, C_1 (Fig. 50), is obtained by adding the ordinates of the current curve in Fig. 49 to those of another curve, exactly opposite in phase to the secondary current, and of such a value as to produce an equal magnetising effect.

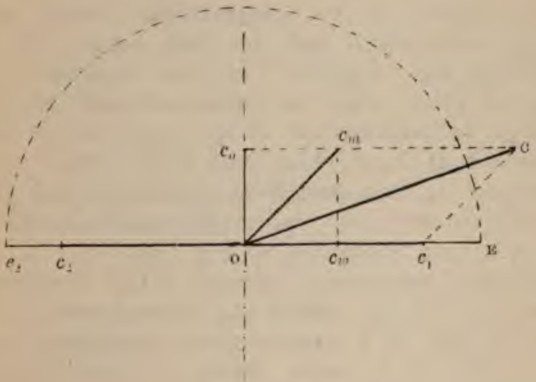

FIG. 51.

(46) **Vector Diagram of Transformer without Leakage.** — In Fig. 51 let $\overline{O\ e_2}$ represent the secondary E.M.F. Then, on the assumption that the voltage drop in primary, due to ohmic resistance,

is negligible — which assumption is generally permissible—the primary impressed E.M.F., E, will be exactly opposite in phase to the secondary or induced E.M.F., and O E will be equal to $O\ e_2$ multiplied by the ratio of turns in the two windings. But, since we have assumed — for uniformity and convenience of construction—that the transforming ratio is 1 : 1, the length O E must be made equal to $O\ e_2$.

With regard to the magnetising current in the primary coil, which will be very small, this will consist of the "wattless" or true exciting current $\overline{O\ C_0}$, in phase with the induction—and, therefore, 90 deg. in *advance* of $\overline{O\ e_2}$ — and the "work" component, $\overline{O\ C_w}$, in phase with $\overline{O\ E}$. This "work" component is due partly to hysteresis and partly to eddy currents, as was explained in Chapter II., article 23, and illustrated by Fig. 19.

The total magnetising current, $\overline{O\ C_m}$, may now be drawn; it will lag behind the impressed E.M.F. by about 45 deg. in a well-designed transformer, which corresponds to a power factor on open secondary circuit of about ·7.

Effect of Closing Secondary on Non-inductive Load.—The load being non-inductive, the secondary current, $\overline{O\ C_2}$ (Fig. 51), will be in phase with the secondary E.M.F. It will be balanced by a component, $\overline{O\ C_1}$, of the primary current, exactly equal and opposite to $\overline{O\ C_2}$ (and therefore in phase with E); and the total primary current will now be represented by the resultant $\overline{O\ C}$.

Effect of Closing Secondary on Partly Inductive Load.—In Fig. 52 let $\overline{O\ e_2}$ be the secondary E.M.F. as before, and $O\ C_2$ the secondary current, which now lags somewhat behind this E.M.F. The balancing component of the primary current will still be equal and opposite to $\overline{OC_2}$, with the result that the primary current, $\overline{OC}$, will also lag behind the impressed E.M.F. It will be evident from inspection of the diagram that the energy put into the primary is still in excess of the energy taken out at the secondary terminals by the amount lost in hysteresis and eddy currents in the core.

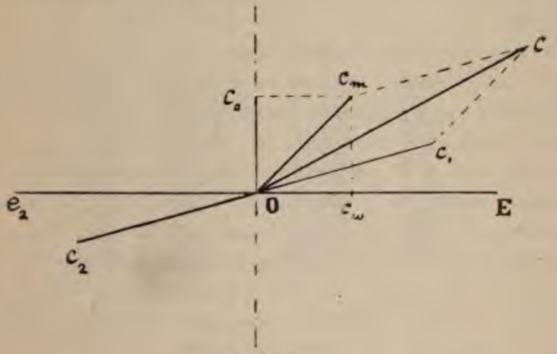

FIG. 52.

(47) **Current Transformers** —The small transformers used in connection with alternating-current measuring instruments to obviate having to take the main current directly through the series coils of the instruments, are known as current or series transformers (to distinguish them from pressure or shunt transformers).

They are used either because the main circuit is a high-potential one, or because the current in same is very heavy, in which case—by suitably arranging the ratio of primary to secondary turns—any convenient current can be passed through the main coil of the instrument.

Such transformers, therefore, play the same part in alternating-current circuits as the heavy current shunts of direct-current instruments; but they have the additional advantage of keeping the instrument connections entirely separate from the main circuit, so that, by suitably insulating the windings of the transformers, the instrument may be made quite safe to handle, even when used on very high-pressure systems.

A current transformer does not differ in principle from a pressure transformer; and the foregoing diagrams are equally applicable to either type. Some care is required in designing these transformers, as it is important that the secondary current should bear, as nearly as possible, a constant relation to the primary current. It can never be theoretically exactly proportional to the primary current, because of the magnetising component of the latter, which does not depend upon the actual value of the total current.

It is important that the necessary magnetising current be kept within small limits, otherwise, on small loads, the proportionality between primary and secondary currents would no longer exist.

It is well to bear in mind that a transformer with its primary connected across constant-pressure mains must never have its secondary short-circuited, otherwise both windings will burn up.

A series transformer, on the contrary, must never have its secondary open circuited, otherwise the induction in the core may reach its saturation limit, and lead to overheating of the iron.

(48) **Polyphase Transformers.** — On any polyphase system, it is always possible to use two or more single-phase transformers, suitably connected up, for the purpose of raising or lowering the pressure of the supply leads.

The use of single-phase transformers has much to recommend it, as, in the event of a breakdown, the repairs are generally more quickly and more economically carried out.

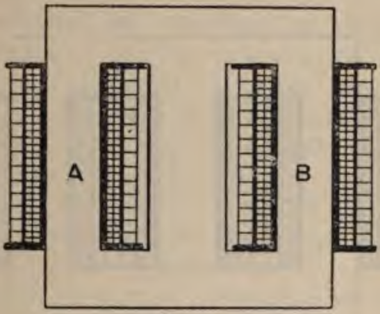

Fig. 53.

Indeed, for large outputs, it is customary to use separate single-phase transformers on the various phases. A polyphase transformer, built up with a common iron core, might become very large; thus, on a three-phase circuit, it would be required to deal with three times the output of each single-phase transformer, and, apart from the difficulties of manufacture and handling, there would be more difficulty in providing adequate cooling surface, with the result that the saving in cost of material required would be very small

In the case of transformers for small outputs it is, however, cheaper to arrange the windings on a common iron core. Figs. 53 and 54 show sections through a two-phase and three-phase transformer respectively.

In the former, the primary and secondary coils belonging to phase A are wound on the left-hand limb of the closed iron circuit, while phase B is wound on the right-hand limb; these two limbs being of equal section. In the centre, between the two bobbins or sets of coils, a common path for the return magnetism is provided, and since this will have to carry either the sum or the difference of the two equal alternating fields having a phase difference of 90 deg., the centre core—if designed for the same flux density as the two outer cores—must have a cross-section $\sqrt{2}$ times as great as either A or B.

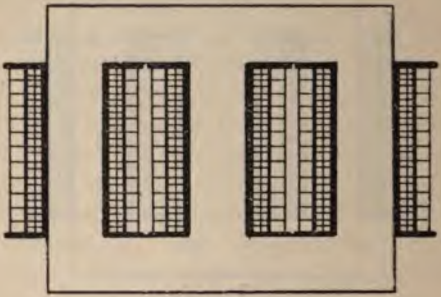

FIG. 54.

In Fig. 54 the three iron cores are equal, and they are each wound with the primary and secondary coils belonging to one of the three phases. With this arrangement in place of three separate transformers, if the flux density is supposed to be the same in each case, the saving of iron in the magnetic circuit will be seen to be quite appreciable: it will be noticed that each of the three iron cores forms the return path for the magnetic flux in the other two, in the same manner as the three conductors of a balanced three-phase circuit suffice to carry the current to and from the apparatus constituting the load.

(49) **Methods of Connecting Three-Phase Transformers** — If it is desirable or necessary to have the neutral point available, the secondary windings of the three-phase transformers must be Y connected; and for small transformers—in which the space occupied by insulation is large in proportion to the cross-section of the copper in the windings—a saving in cost and weight is effected by adopting this mode of connection.

The chief advantage of connecting up the windings in Δ fashion, more especially when three separate transformers are used, is that, in the event of one of the transformers in a group of three breaking down, it can be entirely cut out of circuit, leaving the two remaining sound transformers to provide the necessary three-phase pressure; but this will not be possible if the windings are star connected, since, with this arrangement, any two windings taken together will only provide the pressure across one phase.

With the Δ connection, the removal of one side of the mesh virtually converts the arrangement into a two-phase system having a common return carrying the same amount of current as each of the other two conductors, instead of carrying a current $\sqrt{2}$ times greater, as would be the case with the more usual two-phase system in which the phase angle is 90 deg.

It is not necessary that the primary windings should be connected up in a similar manner to the

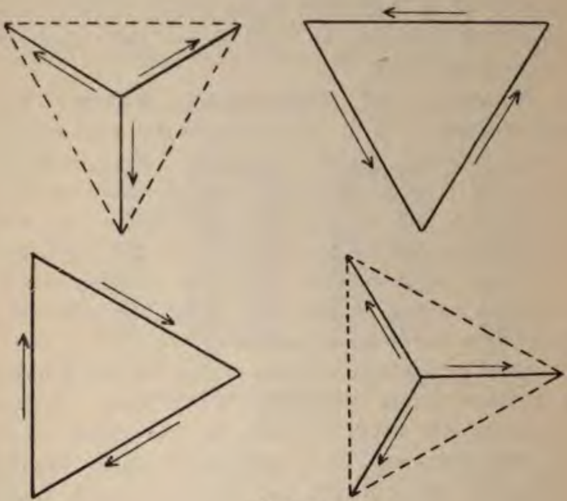

FIG. 55.

secondary windings, but if the primary is star connected and the secondary Δ connected, or *vice versa*, the ratios of the number of turns in the primary and secondary coils will no longer correspond with the transforming ratio as measured across the phases.

The diagrams in Fig. 55 will make this clear.

Here the upper system of vectors indicates the amount and direction of the pressures in the primary coils, while the lower set of vectors indicates the pressures on the secondary side.

The left-hand combination refers to a star-connected primary and a delta-connected secondary, the ratio of transformation being $\frac{1}{\sqrt{3}}$, or ·577 times the ratio of turns; while in the right-hand combination we have a Δ-connected primary and a Y-connected secondary; the transforming ratio in this case being $\sqrt{3}$, or 1·732 times the ratio of secondary to primary turns.

It is interesting to note that, in both these combinations of windings, the secondary pressures *as measured between the three conductors* are not 180 deg. out of phase with the primary pressures (as would be the case if the windings were connected up similarly on both high-tension and low-tension sides), but differ in phase by a right angle, or 90 deg.

(50) **Efficiency of Transformers.**—The efficiency of the alternating-current transformer is very high; its actual value will depend, to a certain extent, upon the skill and knowledge of the designer, but more especially upon the amount of material used in its construction.

The efficiency should never be considered apart from first cost, or without reference to the efficiency as a whole of the system in connection with which the transformers are to be used.

It must also be borne in mind that the temperature rise is no indication of the amount of power lost in the transformer; a transformer which gets very hot may be more efficient than another which keeps comparatively cool. Temperature rise must only be considered with reference to the effect it may have upon the materials used in the construction of the transformer; it is a question which concerns the manufacturer.

The efficiency of a small transformer will necessarily be lower than that of a larger transformer. The maximum efficiency will generally be reached at from three-quarters to full load, and it is important that all transformers on a given system be arranged so that the hours during which they supply a *light* load may be as few as possible.

As approximate figures for well-designed transformers, whether single or polyphase, ·95 to ·96 may be taken as the full-load efficiency of a 3-kw. transformer, while if the output is ten times as great (say 30 kw.), the efficiency would be from ·97 to ·98 at full load.

(51) **Phase Transformation** — In the last chapter reference was made to the principle of replacing the usual three-phase triangle of pressure vectors by any three *star* vectors radiating from a common point O, and terminating at the vertices of the triangle.

Such a system of vectors will produce exactly the same potential differences between the three terminals as the original mesh connection which it is designed to replace.

Scott's System.—The above principle has been made use of by Mr. C. F. Scott in his ingenious arrangement for changing from two to three phases by means of static transformers only.

In Fig. 56 the left-hand diagram shows the usual three-phase triangle of vectors, which may be replaced by the three vectors $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$, all radiating from the common point O. This point lies on the centre of the line A C, and the line O B is, therefore, at right angles to A C. In this manner the three vectors $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ can be replaced by two vectors $\overline{OA}$ and $\overline{OC}$ exactly equal and opposite, and a third vector $\overline{OB}$ having a phase difference of 90 deg. with either of the other two.

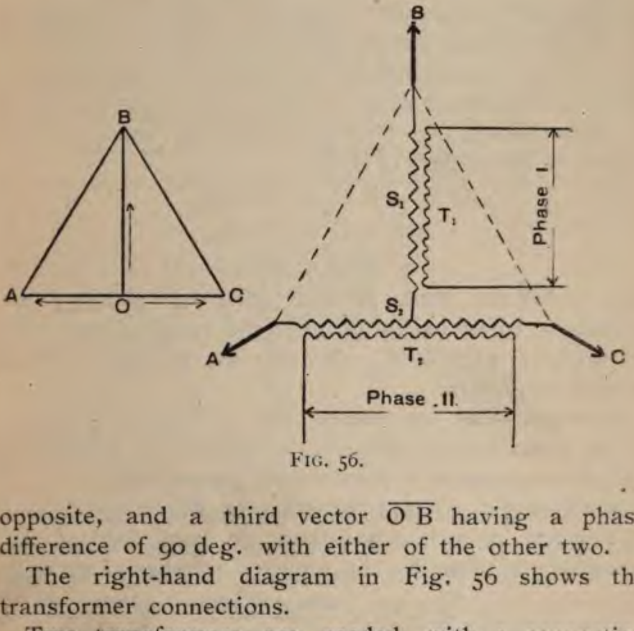

FIG. 56.

The right-hand diagram in Fig. 56 shows the transformer connections.

Two transformers are needed, with a connection from the end of one secondary to the centre point of the other secondary.

If the primaries of the transformers, T_1 and T_2, are fed respectively by phase I. and phase II. of a two-phase supply having a phase angle of 90 deg., the pressures generated in the transformer secondaries, S_1 and S_2, will also be 90 deg. out of phase. It is, therefore, merely necessary to so proportion the number of turns in the windings that the combination of these E.M.F.'s will produce equal pressures between the pairs of terminals A B, B C, and C A. If the transforming ratio of transformer T_2 is 1 : 1, the pressure measured across A C will be the same as the two-phase supply pressure; and if the ratio of turns in transformer T_1 were also 1 : 1, the pressure $\overline{O\,B}$ (see left-hand diagram, Fig. 56) would be equal in amount to $\overline{A\,C}$. This would not be correct: the length of the vector $\overline{O\,B}$ is

$$\begin{aligned} O\,B &= B\,C \sin 60 \text{ deg.} \\ &= A\,C \sin 60 \text{ deg.} \\ &= A\,C \times \cdot 866, \end{aligned}$$

and it follows that, if the ratio of primary to secondary turns in T_2 is 1 : 1, the ratio of turns in T_1 must be 1 : ·866 in order that the triangle A B C may be equilateral.

Example.—Let us assume the two-phase pressure to be 2,000 volts, which it is desired to transform into three-phase at a pressure of 30,000 volts.

Suppose the primary turns on transformers T_1 and T_2 are 200 in each case; then, on *each half* of the secondary winding S_2, the required number of turns is

$$\begin{aligned} \frac{S_2}{2} &= \frac{1}{2} \times \frac{200 \times 30{,}000}{2{,}000} \\ &= 1{,}500 \end{aligned}$$

while the secondary turns on T_1 must be

$$\begin{aligned} S_1 &= \frac{200 \times 30{,}000}{2{,}000} \times \cdot 866 \\ &= 2{,}600 \end{aligned}$$

Lunt's System.—The system of phase transformation described above is reversible—that is to say, it can be used, if desired, for changing from three to two phase.

Another method of effecting this latter transformation is due to Mr. A. D. Lunt. The connections of the two transformers are shown in the left-hand diagram of Fig. 57, while the right-hand diagram shows the vectors of the various *magnetic fluxes* in the cores of the transformers.

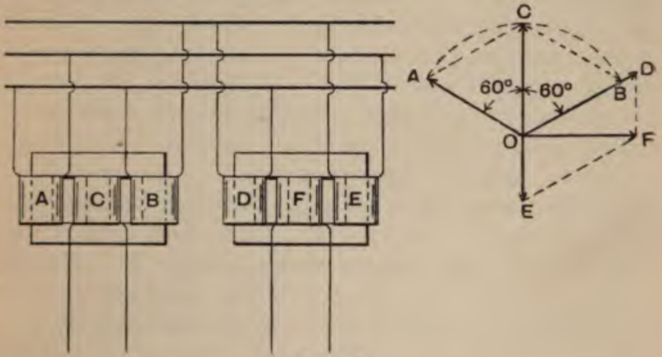

FIG. 57.

Considering first the left-hand transformer, there will be an equal number of turns in the windings A and B, producing equal magnetic fluxes, $\overline{O\ A}$ and $\overline{O\ B}$, differing in phase by 120 deg. The resultant flux through the central core will be $\overline{O\ C}$.

With regard to the right-hand transformer, the number of turns in the coil D will be *smaller* and in the coil E *greater* than in coils A or B, the actual number of turns being such as to produce magnetic fluxes in the cores D and E, of the second transformer, proportional to the lengths of the vectors $\overline{\mathrm{O\ D}}$ and $\overline{\mathrm{O\ E}}$. These vectors must necessarily subtend a phase angle of 120 deg.; but, owing to their *lengths* being suitably proportioned, they will produce a resultant flux, $\overline{\mathrm{O\ F}}$, through the central core, F, exactly at right angles to the flux $\overline{\mathrm{O\ C}}$, in the core C of the first transformer.

It follows that a two-phase supply can be taken from a couple of coils, having the same number of turns, wound on the central cores of the two transformers.

CHAPTER VI.

POWER TRANSMISSION BY POLYPHASE CURRENTS.

(52) One of the chief advantages of polyphase currents is that they make it possible to transmit electric power to great distances. It is probable that, in the future, the continued improvements being made in the single-phase alternating-current motor with commutator, will lead to power being transmitted to considerable distances by means of single-phase currents; but at present—owing partly to the fact that the single-phase induction motor is not self-starting, and partly to the greater efficiency of transmission with three-phase currents — the majority of electric power schemes are carried out with three-phase currents.

It is proposed, however, to consider in succession single, two, and three phase transmission; the former being dealt with somewhat fully, as a clear understanding of the conditions to be dealt with in a single-phase scheme will make the study of polyphase transmission comparatively easy.

(53) **Losses in Transmission.** — With high voltages, such as are necessary for the economical transmission of electric power to a distance, we have to consider, not only the losses due to the heating of the conductors by the current, but also the power lost in the dielectric forming the insulation of the cables, or, in the more usual case of overhead lines,

the discharge at the surface of the conductors, and the waste of power in the air. This loss of power, which is due to discharges from the surface of the conductors, is not — in the present state of our knowledge — easy to predetermine; but it will be greater with high pressures, and conductors of small diameter; it will also depend largely upon the wave form of the alternating E.M.F., and to a certain extent upon the distance between the conductors, and the frequency. It is true that exceptionally high pressures have to be applied before a visible discharge to the air takes place from the surface of the conductor—probably between 100,000 and 200,000 volts for a wire ·5 in. diameter—but it does not follow that there is not an appreciable leakage taking place at very much lower pressures, especially in the case of small wires. In addition to these two causes of power dissipation, we must not overlook the leakage over insulators on a long high-tension aerial line; but the amount of current which leaks to earth, or between conductors, in this manner is very small on a carefully-designed and well-constructed line, and it is generally permissible to neglect it. The allowable loss of power in a transmission line is a matter of the greatest importance, and it will be again referred to after we have considered the various systems and arrangements of conductors; but for the present a certain percentage loss in transmission will be assumed, apart from economical considerations.

(54) **Choice of Voltage.**—Generally speaking, the higher the pressure the more economical will be the transmission, because, for a given total power, the current, and, therefore, the weight of the con-

ductors, is reduced. But there are, obviously, many considerations which stand in the way of very high pressures being used in all cases; and each particular scheme must be examined from every possible point of view before definitely deciding upon the voltage to be adopted. In the first place, a small current at a very high pressure costs more to produce than a larger current at a proportionately lower pressure, either because the generators themselves will be more costly, or because step-up transformers will have to be used. If, however, the line losses would be very considerable with the larger currents, then it might be more economical to lay out capital in step-up transformers, extra insulation of conductors, and step-down transformers at the receiving end, and so save copper in the lines. In other words, the most suitable voltage will depend very largely upon the *distance* to which the power has to be transmitted. Indeed, Mr. C. F. Scott has suggested as a rough-and-ready rule that the pressure in thousands of volts should be taken as equal to about one-third of the total distance in miles. Thus, a pressure of 10,000 volts would be suitable for a 30-mile transmission.

It is doubtful whether this rule would be applicable to the case of very long transmissions, because the difficulties of dealing with very high pressures would be enormous. As an example: for a line 300 miles long—which is by no means an unlikely distance to be practically dealt with—the pressure, according to Mr. Scott's rule, would be 100,000 volts. This is not an impossible pressure, although 80,000 volts have not been exceeded up to the present. At the same time, it is very unlikely that, for dis-

(56) **Conductor Impedance.**—Imagine a straight length of cable of fairly large cross-section, through which a steady continuous current is flowing. The lines of magnetic induction due to this current will not all pass through the non-conducting medium surrounding the conductor, but a certain number of them—due to the current in the central portions of the cable—will pass through the substance of the conductor itself. In other words, the magnetic flux surrounding one of the central strands of the cable will be greater than that which surrounds a strand of equal length situated near the surface. It follows that, if the circuit be now broken, the current will die away more quickly near the surface of the conductor than at the centre; and, for the same reason, on again closing the circuit the current will spread from the surface inwards.

If, now, we imagine the conductor to be used for conveying an alternating current, it is evident that, with a sufficiently high frequency (or even with a low frequency if the conductor be of large cross-section) the current will not have time to penetrate to the interior, but will reside chiefly near the surface of the cable. This crowding of the current towards the outside portions of the conductor has the effect of apparently increasing its resistance; and it follows that, if C is the total current in the cable and r its ohmic resistance, the power lost in watts would no longer be $C^2 r$, as would be the case if the current were a steady one; but $C^2 r'$, where r'—which stands for the apparent resistance of the conductor—is greater than r, its true resistance This effect has been very carefully investigated by Lord Kelvin and others, and it is from the data obtained from their

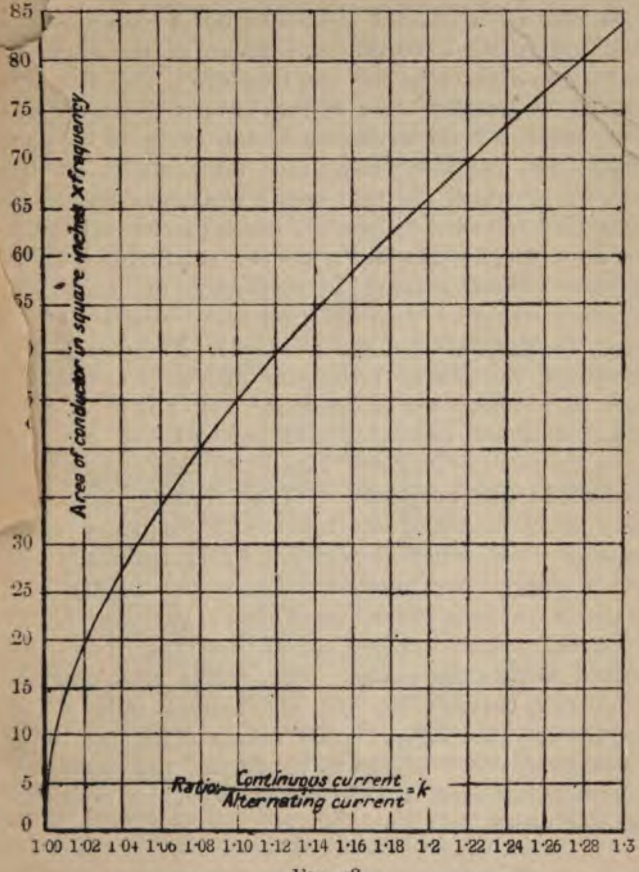

FIG. 58.

calculations that the curve in Fig. 58 has been drawn.

It is found that, for any given apparent increase in resistance of a circular conductor due to the une distribution of the current, the square of the diameter must vary inversely as the frequency, and for this reason the product *area of conductor* × *frequency* has been taken for the ordinates of the curve in Fig. 58, while the ratio k, measured horizontally, is the number by which the true ohmic resistance has to be multiplied in order to give us the apparent resistance. As an example, consider a copper conductor $\frac{3}{4}$ in. in diameter, and assume a frequency of 50. The product sectional area × frequency works out at 22·1, corresponding to a multiplier $k = 1·025$. If, therefore, the ohmic resistance of the transmission line is r ohms per conductor, the power lost in overcoming the resistance will not be $C^2 \times 2\,r$, but $C^2 \times 2\,r'$, where $r' = r \times 1·025$. In this example the increased loss of power with alternating current is very small, and had we taken the frequency at 30 instead of 50, which would not be exceptionally low for a power transmission scheme, the multiplier k would have been almost negligible; but with higher frequencies and large conductors the correction should always be made. The values of k obtained from the curve (Fig. 58) are correct only if the conductor is of *copper*. To obtain k for any other "non-magnetic" cylindrical conductor, the product of the sectional area and frequency must be multiplied by the ratio

$$\frac{\text{conductivity of metal of conductor}}{\text{conductivity of copper}}$$

before obtaining the corresponding value of k from the curve. If the conductor is of iron (or other "magnetic" material), the value of k may be much greater than the value read off the diagram.

It is a not uncommon belief that when aluminium conductors are used in place of copper there will be a greater increase of resistance to alternating currents than when the conductors are of copper, because of the greater diameter of wire required to give the same conductivity; but the above multiplying ratio makes it clear that the percentage increase of losses with alternating currents of the same frequency will be independent of the material of the conductor (iron excepted), because the greater sectional area necessary to maintain the same ohmic resistance of the lines when a wire of lower conductivity is used is evidently exactly balanced by the higher specific resistance of the metal.

(57) **Effect of Inductive Load on Line Losses.**—Let us still assume the line to be without self-induction or capacity, and see what is the effect on the line losses if the "load" at the distant end has a power factor less than unity, such as would be the case if the current is supplied to induction motors. Assuming a lag of 37 deg., which corresponds to a power factor (*cos* 37 deg.) of about ·8, the total current, to deliver the same power at the same pressure, will be 1·25 times greater than if the power factor were unity; and since, for the same loss of power in the line, the resistance multiplied by the square of the current must remain constant, it follows that the cross-section of the conductors must vary inversely as the *square* of the power factor, the proper correction being made for increased *conductor impedance* if of sufficient importance. In the case under consideration, the cross-section would

have to be increased in the ratio of 1 to ·64, which corresponds to an increase in weight of copper of over 50 per cent. It does not follow that it would be economical to increase the section to so great an extent, but the importance of a high power factor must not be overlooked.

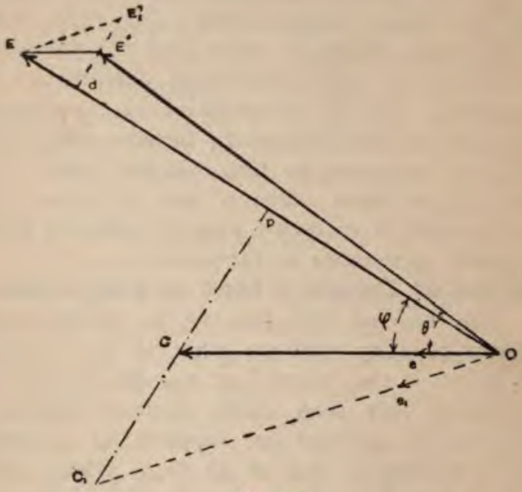

FIG. 59.

In the vector diagram (Fig. 59) $\overline{OE'}$ is the voltage at receiving end, and $\overline{OC}$ the current, lagging behind E′ by an angle θ; $\overline{Oe}$ is the loss of pressure in the line (equal to $2r \times C$, or to $2r' \times C$ if the correction for conductor impedance is made), and $\overline{OE}$—obtained by compounding E′ and e—is the initial pressure at generating end. A study of this diagram leads to the conclusions (1) that the power factor at the generating end is greater than at the receiving end, and (2) that, *for a definite percentage* $C^2\,r$ *loss in the line*, the actual difference between the pressures at the two ends of the line (represented by the distance E *d*) will become *less* as the load becomes more inductive—*i.e.*, as θ is increased. If, on the other hand, we let the losses in the line take care of themselves—that is to say, if we consider a line of *definite resistance receiving a definite total amount of real power at the generating end*, and examine the effect of variable power factors, we shall find (3) that for comparatively small percentage losses in the line the actual difference, E *d*, between the pressures at generating and receiving ends is practically constant, and independent of the power factor. The dotted lines in Fig. 59 will make this clear. Assuming the initial pressure, $\overline{\text{O E}}$, to remain constant, the point C of the current vector, $\overline{\text{O C}}$, must lie on the perpendicular—*p* C to the line O E, whatever may be the power factor (since the *power*, $E\ C\ \cos\phi$, is constant): this means that the point, E′, of the vector representing the *receiving end* pressure must lie on the perpendicular *d* E′ to the line O E, the triangles O C *p* and E E′ *d* being similar. Moreover, since the length of the perpendicular, *d* E′, or *d* E'_1, is small in comparison with the length O *d*, it differs very little from the arc of a circle described through *d* from the centre, O, and, therefore, the difference between $\overline{\text{O E}}$ and $\overline{\text{O E}'}$ is practically constant under the conditions stated above.

(58) **Effect of Taking into Account the Self-Induction of the Lines.**—Since there must be a certain distance separating the two wires forming the go and return of the alternating-current transmission line, there must of necessity be an E.M.F. of self-

induction produced by the alternating magnetic flux in the space between the wires, which will be directly proportional to the amount of current in the line.

In Fig. 60, $\overline{O\,E'}$ is the pressure at receiving end, and $\overline{O\,C}$ the current, lagging behind by an angle θ, exactly as in Fig. 59; but now, instead of merely considering the effect of the ohmic drop, $\overline{O\,e}$, we must also take into account the back E.M.F. due to the self-induction of the transmission lines. This E.M.F. will lag behind the current by a quarter of a period, and the vector, $\overline{O\,L}$, has been drawn 90 deg. *in advance* of the current to represent the component of the total pressure at generating end

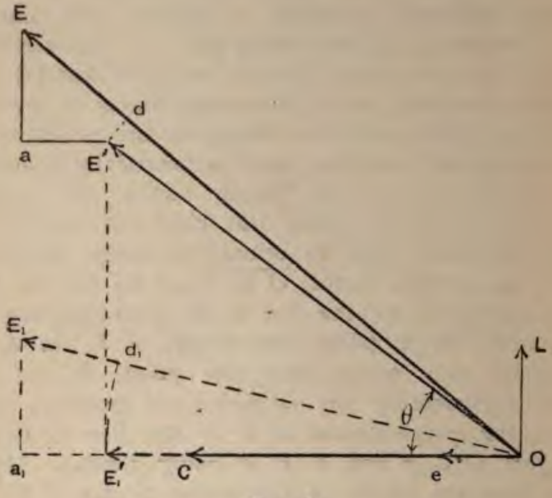

FIG. 60.

necessary to balance the E.M.F. of self-induction. The total requisite initial pressure is evidently $\overline{\text{O E}}$, obtained by compounding the forces $\overline{\text{O E}'}$, $\overline{\text{O }e}$, and $\overline{\text{O L}}$. The dotted lines show the effect of resistance drop combined with inductive drop for the same current in the line, and the same total amount of power transmitted, when the power factor at receiving end is unity—*i.e.*, when $\theta = 0$.* A study of this diagram (Fig. 60) leads to the following conclusions:

(1) The additional loss of pressure due to the self-induction of the lines is of considerably greater importance on an inductive than on a non-inductive load.

(2) For a given current and $C^2 r$ loss in the line, the difference in the power factors at the two ends is greatest when the load is non-inductive.

(3) On a partly inductive load there is a certain value of the self-induction which, at a given frequency, will make the power factors at the two ends of the line equal—*i.e.*, it will bring $\overline{\text{O E}}$, in Fig. 60, to coincide with $\overline{\text{O E}'}$. If the self-induction is *greater* than this particular value, the power factor at generating end will be *less* than at receiving end, and *vice versa*.

(59) **Predetermination of Inductive Drop.** — In the diagram Fig. 60 we have assumed the inductive pressure drop—represented by the vector $\overline{\text{O L}}$—to be known. In order to calculate it, we must know the coefficient of self-induction of the

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* This method of drawing the diagrams, which assumes the current constant and the pressure variable, to comply with the condition of constant *power* at different power factors is sometimes convenient, but the construction can readily be modified to suit the condition of a definite initial pressure.

circuit formed by the go and return wires. This will depend not only upon the distance separating the wires, but also upon their diameter. The greater the distance between wires, and the smaller their diameter, the larger will be the amount of magnetic flux produced by the passage of unit current.

The curve drawn in Fig. 61 is plotted from figures published by Messrs. Houston and Kennelly in an article which appeared in the *Electrical World* a few years ago. It gives the coefficient of self-induction per mile of *single conductor* ·4 in. diameter in millihenrys.* If the diameter of the wire is not ·4 in., the coefficient of self-induction must be read off the curve, not to correspond with the actual distance, a, between the wires, but for an imaginary distance, a', such that

$$a' = a \times \frac{4}{d},$$

where d = the actual diameter of the wire, in inches.

The well-known formula for the E.M.F. of self-induction (see Note 2, at end of book), *on the assumption of the wave forms being sine curves*, is

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* The calculations are based on the assumption of an even distribution of current throughout the section of the wire, such as would occur if continuous currents were transmitted; but when the applied E.M.F. is a rapidly alternating one, the current does not utilise the central portions of the conductor to the same extent, and although the flux of induction *outside* the bounding surface of the conductor will be the same as before, the lines which pass through the space occupied by the material of the conductor are now fewer than in the first instance. The result is that, in the case of comparatively high frequencies, the figures obtained from the curve will lead to a calculated inductive drop somewhat *higher* than would actually be obtained. It is, however, usually unnecessary to apply any correction on this account, seeing that the wave form of the alternating current cannot be exactly predetermined, and the results of calculation are, at the best, but a more or less close approximation to the results obtained in actual working.

$$e = 2 \pi n \mathrm{L} \mathrm{C},$$

where n = the periodicity;
L = the coefficient of self-induction in henrys;
C = the current in amperes,

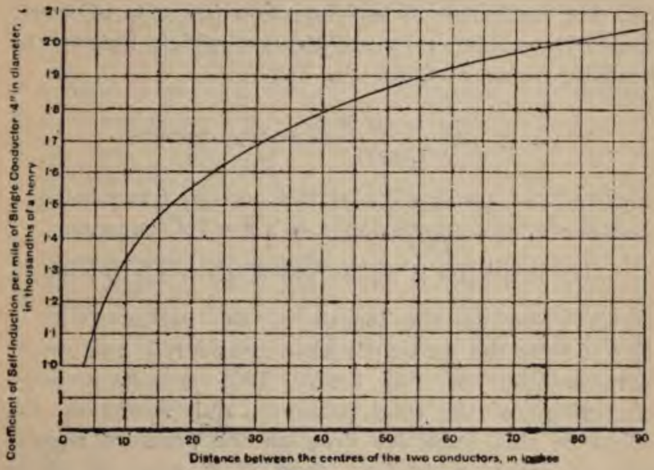

FIG. 61.

and we are now in a position to calculate the value of the vector $\overline{\mathrm{O\,L}}$ in Fig. 60.

Example.—Consider an overhead transmission line delivering a total power of 1,000 kw. a distance of 50 miles with a pressure of 20,000 volts at a periodicity of 25. The current (if we suppose the power factor to be unity) will be 50 amperes, and we shall assume the conductors to be ·36 in. in diameter, separated by a distance of 3 ft. In the first place, the correction to be made for the diameter of wire, before referring to the curve Fig. 61, gives us

$$a' = 36 \times \frac{\cdot 4}{36} = 40 \text{ in.},$$

and, taking this figure as the distance between wires, instead of the actual distance of 36 in., we find the coefficient of self-induction per mile of single conductor to be 1·79 millihenrys, which makes the probable value of L for the line

$$L = \frac{1{\cdot}79 \times 50 \times 2}{1{,}000} = \cdot 179 \text{ henrys},$$

and, on the assumption of the current wave being a sine curve, the expression $e = 2 \pi n \text{ L C}$ becomes

$$e = 2 \times 3{\cdot}1416 \times 25 \times \cdot 179 \times 50 = 1{,}410 \text{ volts},$$

which gives us the length of the vector $\overline{\text{O L}}$ in Fig. 60 for the particular case considered and under the condition of full load. This is not a large percentage of the total pressure, and, moreover, the effect of this inductive drop on the pressure regulation is considerably less than if it were in phase with the supply pressure; but with large currents, high frequencies, and low power factors, the loss of volts due to the self-induction of the lines may become a very serious matter.

(60) **Capacity of Transmission Lines.**—In the case of a concentric or other underground cable conveying alternating currents, the capacity effects may be very great. A large capacity is generally objectionable in practical working, although the effects of capacity tend to balance those of self-

induction, and so bring the power factor nearer to unity. If K is the capacity in microfarads of any system of two conductors, n the frequency, and E the potential difference between the conductors, then the *charging current—i.e.*, the alternating current which will pass between them irrespective of the

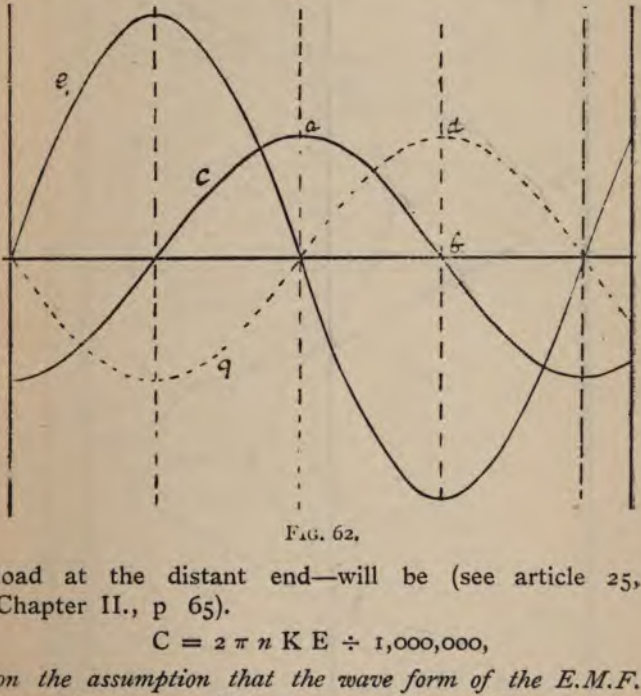

FIG. 62.

load at the distant end—will be (see article 25, Chapter II., p 65).

$$C = 2 \pi n K E \div 1{,}000{,}000,$$

on the assumption that the wave form of the E.M.F. is a sine curve. This current is in quadrature with the E.M.F., being 90 deg. *in advance* of the applied potential difference, or 90 deg. *behind* the condenser E.M.F. (e_1) as shown in Fig. 62.

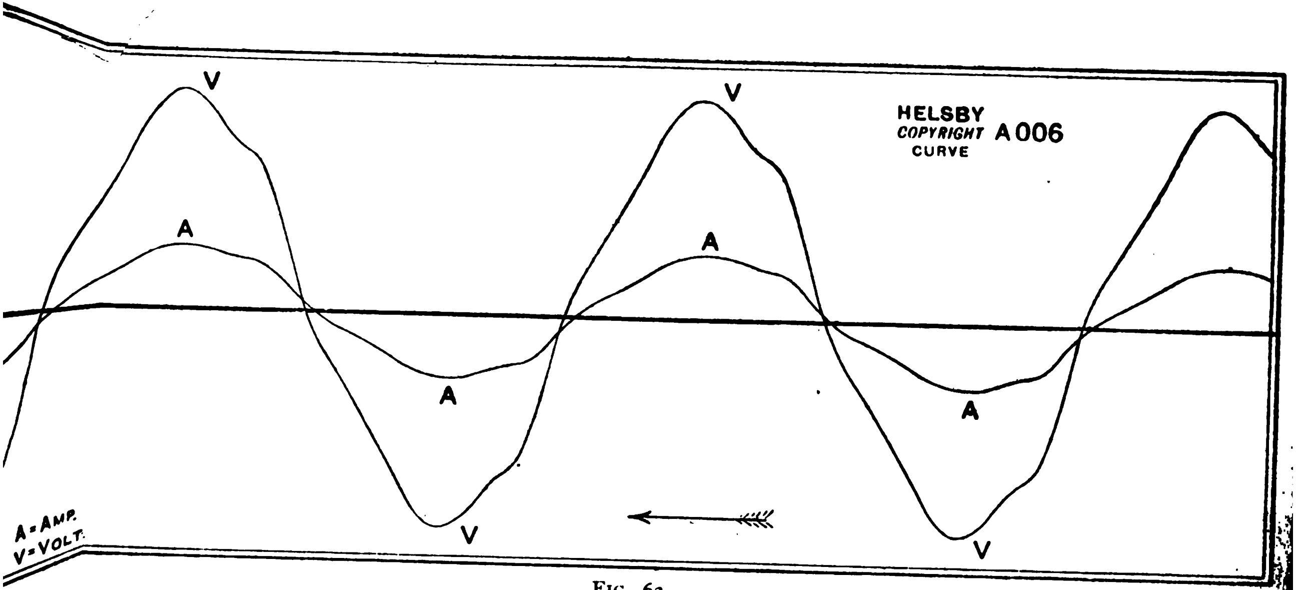

Fig. 63.

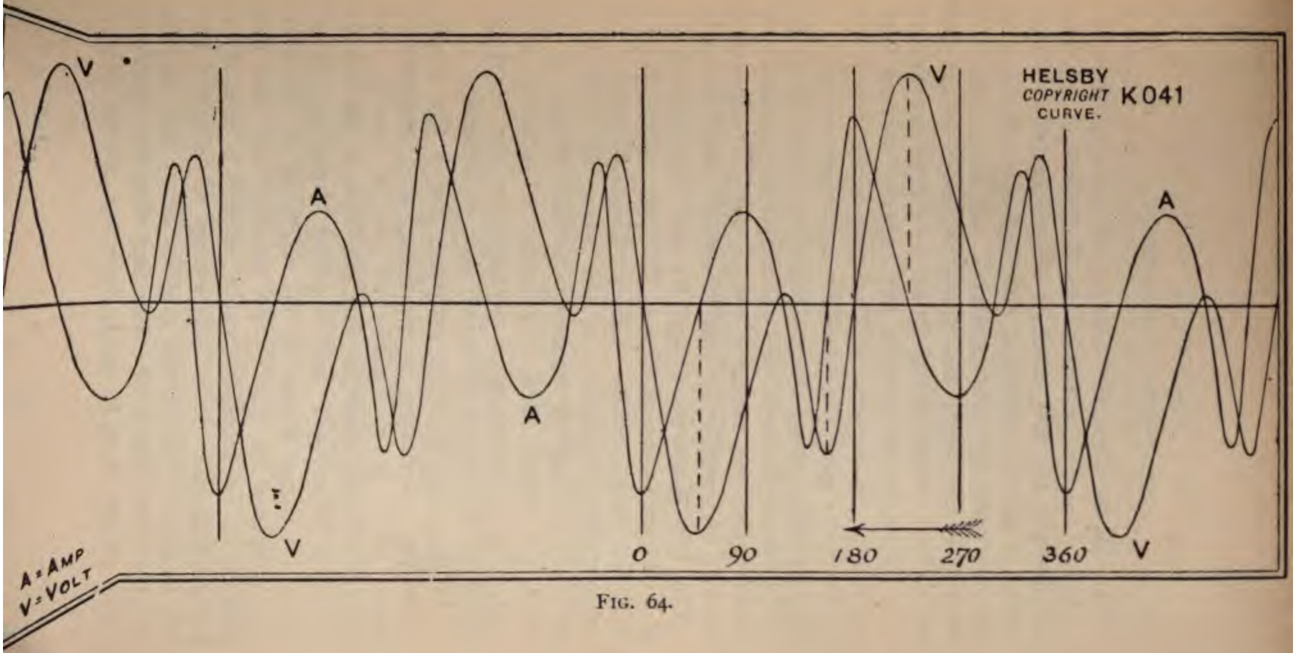
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FIG. 64.

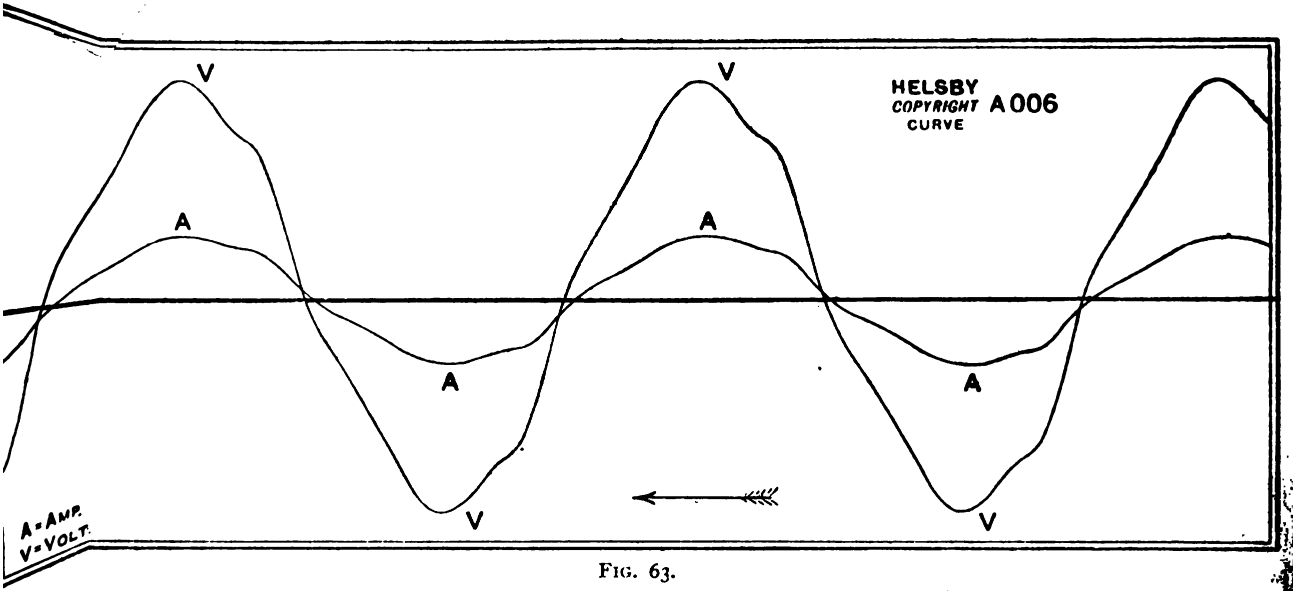

FIG. 63.

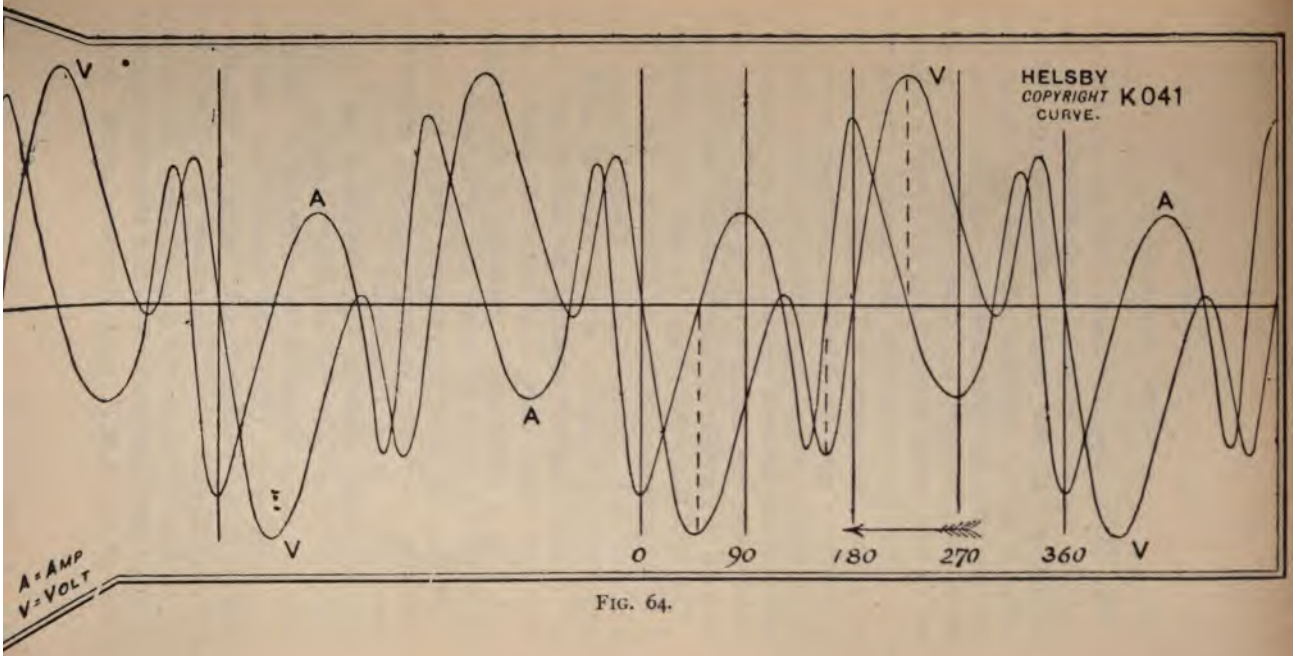
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FIG. 64.

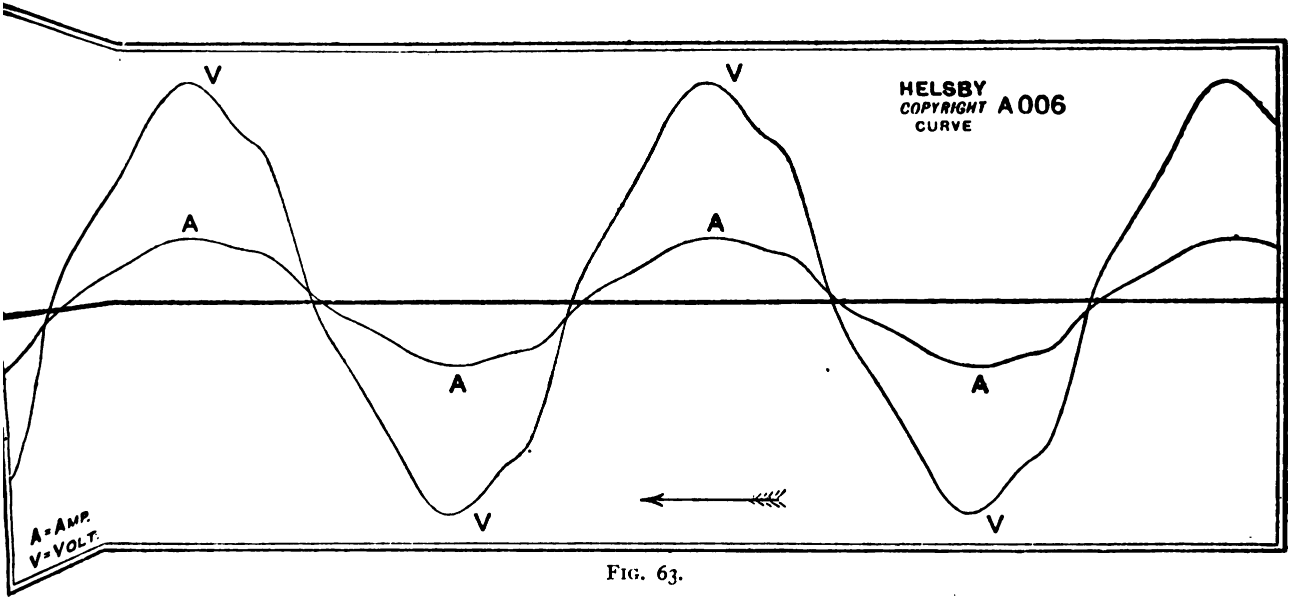

FIG. 63.

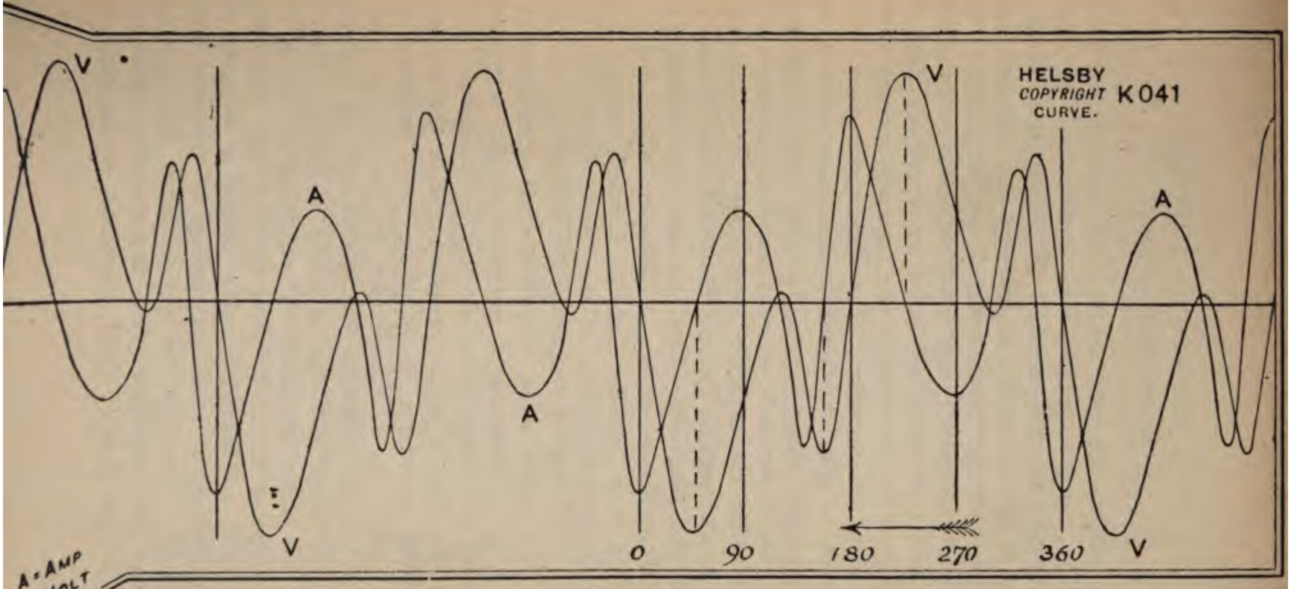
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FIG. 64.

The above formula enables us to calculate the capacity current without any difficulty, on the assumption of the E.M.F. following the simple harmonic law of variation, provided the capacity, K, can be correctly predetermined, or ascertained by actual measurement ; but when we come to consider the various wave forms found in actual practice, it is not possible to calculate the capacity current with any degree of accuracy. The sine curve wave will be found to give the smallest condenser current of any possible wave form, and the most intricate mathematical methods will not enable us to accurately predetermine the capacity effects due to alternating currents of irregular or unsymmetrical wave forms.

The two diagrams (Figs. 63 and 64), reproduced by kind permission of Mr. A. Whalley, of the British Insulated and Helsby Cables, Limited, show the extraordinary distortion of the wave forms due to the introduction of capacity in the circuit of an alternator which does not, under all conditions of load, give a difference of potential at the terminals following the simple harmonic law of variation.

In Fig. 63, V is the pressure and A the current when the alternator is supplying 29·5 amperes at 2,040 volts to a transformer on a nearly non-inductive load, whereas Fig. 64 shows the altered pressure curve and the resulting charging current when the same alternator is supplying a current of 6·04 amperes at 1,980 volts between the two conductors of an insulated cable having a capacity of 4·9 microfarads. It is interesting to observe how the smallest ripple in the original curve may become distorted and magnified. The fact that the maximum *values of the* E.M.F. (and, therefore, of the charge) invariably occur at the instant when the current is changing its direction, should also be noted. (See Chapter II., article 25, and Fig. 62.)

In the case of overhead lines, the capacity effects are generally small, and sometimes negligible. The calculation of the capacities between wires, even when symmetrically arranged, appears to offer considerable difficulties to mathematicians, but the formula

$$K = \frac{0194}{\log_{10} d/r}$$

applies to the case of a single-phase overhead line.*

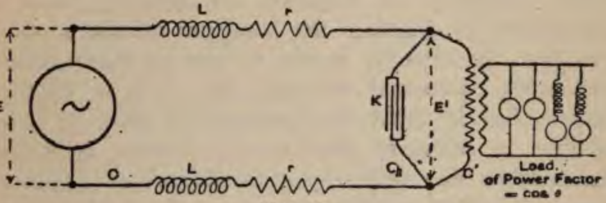

FIG. 65.

Here K = the capacity per mile of circuit in microfarads;
d = the distance between wires;
r = the radius of the wires.

(61) **Vector Diagram of Single-Phase Transmission Line, taking into account Resistance, Self-Induction, and Capacity.**—In Fig. 65, the generator is shown supplying the total current, C, at the initial pressure, E, through the transmission lines to the distant end, where the load is supposed to be partly inductive. The current delivered at the

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* Mr. G. H. Baillie in the *Electrician*, Aug. 8, 1902.

end of the line is C′, and the pressure E′; the coefficient of self-induction of the transmission lines is 2 L, and their resistance 2 r. It will be noticed that the whole of the capacity is shown as being concentrated at the end of the line. This assumption leads to a calculated capacity current somewhat in excess of what would be obtained with the same total capacity distributed along the line, but it simplifies the vector diagram. A closer approximation to actual conditions would be obtained by supposing the capacity to be concentrated at the centre of the line (or, perhaps, rather nearer the receiving than the transmitting end); or if still greater accuracy be required, no difficulty need be experienced in drawing the diagram on the supposition that the capacity is due to two or more smaller condensers connected between the lines at different points. But it must be remembered that it is not possible to calculate the capacity current on a long transmission line with any great degree of accuracy, because it will depend largely upon the E.M.F. wave form, which, even if known under certain specified conditions, will generally vary considerably with alterations in the load. The capacity of a system of overhead conductors will also depend somewhat upon the state of the atmosphere.

Assuming, then, for the sake of greater simplicity of construction that the total capacity, K, of the line is concentrated at the distant end, we may proceed to draw the vector diagram Fig. 66. Here $\overline{OE'}$ and $\overline{OC'}$ represent, respectively, the potential difference and the current at the distant end of the line, the two vectors being drawn with an angle θ between them, such that cos θ is equal to the assumed power factor of the load. Referring to Fig. 65, it will be seen that a condenser, of capacity K, is supposed to connect the wires at the distant end; the impressed volts at condenser terminals are therefore E′, and the vector for the condenser current (equal to $2 \pi n$ K E′, on the assumption of the sine curve wave form) must be drawn 90 deg. in advance of $\overline{\text{O E}'}$. It is represented by $\overline{\text{O C}_k}$ in Fig. 66. The total current supplied to the line at the generator end will be $\overline{\text{O C}}$, obtained by compounding $\overline{\text{O C}'}$ and $\overline{\text{O C}_k}$. With regard to the E.M.F. at generator end, this is the resultant of three components—namely: $\overline{\text{O E}'}$, the pressure available at receiving end; $\overline{\text{E}' a}$ (drawn parallel to O C and equal to C × $2 r$), to represent the drop due to resistance; and $\overline{a \text{ E}}$ (drawn at right angles to O C and equal to $2 n \pi$ × C × 2 L),

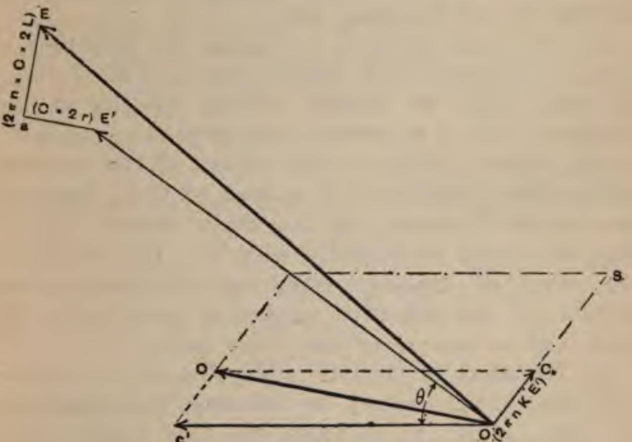

FIG. 66.

to represent the pressure required to balance the E.M.F. of self-induction. This gives us $\overline{OE}$ as the necessary pressure at generating end.

The conclusions to be drawn from this diagram (which takes both self-induction and capacity into account) are as follows:

(1) On an inductive load the current, C, put into the line at the generating end may be *less* than the current, C′, at receiving end.

(2) The total current, C (under the condition of a partly inductive load), comes more and more nearly in phase with the E.M.F. as the capacity current increases up to a certain limit depending upon the power factor of the load. Thus, if the capacity current were equal to O S instead of O C_k, the total current—as indicated by the chain dotted lines in Fig. 66—would be in phase with E′. This illustrates the effect of capacity supplying the magnetising current of the inductive apparatus constituting the load, and so improving the power factor.

(3) The effect of capacity (always on the assumption of a low power factor), by reducing the current in the line, is also to diminish the $C^2 r$ losses, or to permit of a smaller section of conductor being used.

It must not be forgotten that these conclusions are derived from a diagram which represents the conditions under full load, or when the total current is large in comparison with the charging current; and, in considering the question of power losses, we must not overlook the fact that, *at light loads*, the charging current may be the cause of *increased* $C^2 r$ losses. In other words, on a low *load factor* (whatever may be the power factor, but more especially when the power factor is high), a large capacity current may not be an unmixed blessing, even when viewed solely from the point of view of its effect on the line losses. As a matter of fact, the charging current on overhead lines is generally small. Let us take as an example the same data as served us to illustrate the effects of self-induction—that is to say:

The pressure $E' = 20{,}000$ volts.

The frequency $n = 25$.

The length of the line = 50 miles.

The diameter of conductors = ·36 in.

Their distance apart = 3 ft.

Using the formula given above for the capacity of an overhead line, we have

$$\text{capacity in microfarads per mile} = \frac{\cdot 0194}{\log 36/\cdot 18}$$
$$= \cdot 00843,$$

which gives us for the total capacity,

$$K = 50 \times \cdot 00843$$
$$= \cdot 4215 \text{ microfarads.}$$

The capacity current will be

$$C_k = 2\pi \times 25 \times \cdot 4215 \times 20{,}000 \times 10^{-6}$$
$$= 1 \cdot 3 \text{ amperes,}$$

which is a very small percentage of the total current (assumed to be about 50 amperes). At the same time, we have only to suppose an increase in the length of line from 50 to 200 miles, with a correspondingly higher pressure of, say, 50,000 volts, and imagine the frequency to be 50 instead of 25, in order to obtain a calculated capacity current of 26 amperes, which is by no means negligible.

It will be understood that where insulated cables

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are used (only possible for lower pressures and shorter distances), the capacity current may be from twenty to thirty times as great as in the case of an equivalent overhead transmission, and, moreover, the dielectric losses may become important.

(62) **Rise of Pressure at the Distant End of a Long Transmission Line.**—A slight modification of the diagram Fig. 66 will clearly show the well-known effect of a *rise* of pressure occurring at the far end of a line *when the load is very small*, provided also that *both self-induction and capacity are present.* We shall still suppose the arrangement to be as shown in Fig. 65, with the one exception that the transformer and load at the receiving end are entirely disconnected. We shall also suppose the capacity and self-induction to be greater than in the previous example, so as to magnify the effect.

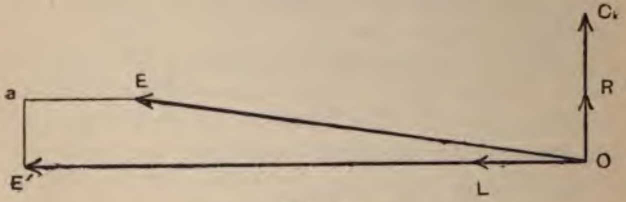

FIG. 67.

Draw $\overline{O\ E'}$ in Fig. 67 to represent the pressure at the distant end; then $\overline{O\ C_k}$ (at right angles to $\overline{O\ E'}$ — in the forward direction — and equal to $2\ \pi\ n$ K E′) is the capacity current which, under the condition of open circuit at distant end, is also the total current flowing in the wires. We now proceed with the construction exactly as in

Fig. 66, making $\overline{E' a}$ (in phase with he current) equal to $C_k \times 2\ r$, and a E (at right angles to the current) equal to $2\pi n \times C_k \times 2$ L. The vector $\overline{O E}$, which, it will be seen, is *smaller* than $\overline{O E'}$, is the necessary pressure at generating end. It will be noted that, if self-induction were absent, the pressure at generating end would be $\overline{O a}$, which is practically equal in length to $\overline{O E'}$; but, since the E.M.F. of self-induction, $\overline{O L}$ (for this particular state of things), is *in phase with E'*, it stands to reason that the necessary pressure at generating end will be *reduced* accordingly

Rises of pressure at the end of long underground feeders are by no means uncommon, and they may also occur on long-distance overhead lines. Indeed, a rise of 8 per cent. has been observed on a 40-mile 25,000-volt line when the circuit was open at the distant end. It should be mentioned that the pressure rises due to the combined effects of capacity and self-induction are often greater in practice than would be indicated by the construction of a diagram such as Fig. 67. But this diagram is based on the assumption of the E.M.F. wave being a sine curve, and the abnormal effects sometimes met with in practice are almost invariably due to peaked or irregular wave forms. These may lead to enormously increased capacity effects, which, unfortunately, cannot be predetermined with any degree of accuracy. The writer does not see that there can be any insurmountable difficulties in the way of designing alternators to give E.M.F. waves approximating to sine curves; careful attention to the arrangement and shape of the pole-pieces should lead to the desired results. That such a wave form is desirable in nearly all cases there can be little doubt, yet few manufacturers of alternating-current generators appear to have given special consideration to the matter. The essential point to bear in mind is not that the wave should exactly coincide with the theoretical sine curve, but that, *especially on open circuit*, it should be free from ripples or peaks.

With regard to *electrical resonance*, this is generally supposed to be responsible for nearly all unlooked for and, at first sight, unaccountable capacity troubles; but exactly what they mean by resonance, few, if any, electrical engineers are able to explain. There are, no doubt, certain scientists and mathematicians who have a clear understanding of the causes which lead to resonance effects; but they have not yet succeeded in expressing these ideas in popular language. The writer prefers, if possible, to avoid any reference to electrical resonance, because of the indefinite if not erroneous ideas which the expression suggests. It would, however, seem as if every transmission line had a certain natural period of wave oscillation, with the result that, when the periodicity of the supply synchronises with it, abnormal pressure rises may occur. This synchronism might occur not only with a definite alternator speed, but also through ripples or irregularities in the wave form; and this is an additional reason why machines giving sine curve waves under all conditions of load would seem to be the ideal generators for the purpose of power transmission.

(63) **Transmission by Two-Phase Currents.**—Let us first assume, as when considering single-phase

transmission, that not only the load at the distant end, but also the transmission line, are without self-induction or capacity.

If we run four conductors as indicated in Fig. 68, keeping the two phases entirely separate, the arrangement resolves itself into the transmission of two distinct single-phase currents; and if the load is equal on both sections, the total power transmitted will be

$$W = 2\,E \times C$$

or,

$$2 \times C^2 \times (R + 2\,r),$$

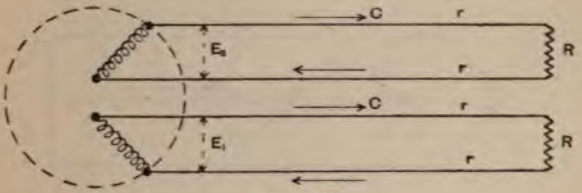

FIG 68.

where E = the terminal pressure of each section and C = the current in each section.

The pressure lost in transmission will be $2\,r \times C$, where r is the resistance of a single conductor; and the necessary weight of copper in the conductor, for a given pressure drop (or loss of power) will be the same as for a single-phase transmission scheme. It is, however, evident that, by combining two of the conductors to form a common return, the transmission of two-phase currents can be effected with only three wires, as shown in Fig. 69.

The vector diagram for such a system of transmission is very simple *if we can assume the resistance, r, of the common conductor to be negligible*, and it has been drawn in Fig. 70. Here the phase difference of a quarter period between the two E.M.F.'s is represened by E_1 being drawn 90 deg. in advance of E_2, and since the circuits are balanced—*i.e.*, since the load is the same on both phases—we may write

$$C_1 = C_2 = \frac{E_1}{r + R} = \frac{E_2}{r + R}$$

and as we are not taking into account either capacity or self-induction, the two currents will be in phase with the respective E.M.F.'s, and the return current in the common conductor will be 45 deg. in advance of E_2 and 45 deg. behind E_1: its value (owing to O C being the hypothenuse of a right-angled triangle) will be $\sqrt{2}$ times C_1 or C_2.

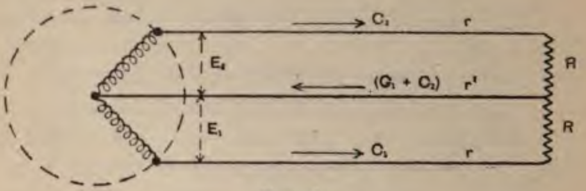

FIG. 69.

We shall now see how the problem is somewhat complicated as soon as the common return conductor has appreciable resistance.

For the purpose of drawing the diagram Fig. 71, we may suppose the E.M.F.'s, E_1 and E_2, at generating end, to be the same as in the previous diagram; but the pressure drop will be greater, owing to the resistance of the common return; and—for the sake of exaggerating the alteration in the diagram—we shall assume this drop (in the middle wire alone) to be as much as half the impressed voltage—*i.e.*, $\frac{1}{2} E_1$ or $\frac{1}{2} E_2$.

Owing to the load being balanced, and the connections symmetrical, the fact that the middle conductor has appreciable resistance will not alter the phase relation of the total current, which will still be midway between the two E.M.F.'s, as in Fig. 70; and since the component of the E.M.F. required to overcome the resistance, r', of the middle conductor must be in phase with this total current, we may draw $\overline{O\ a_3}$ to represent this component in such a direction as to make an angle of 45 deg. with $\overline{O\ E_1}$ and $\overline{O\ E_2}$, and of a length equal to one-half of either of these vectors.

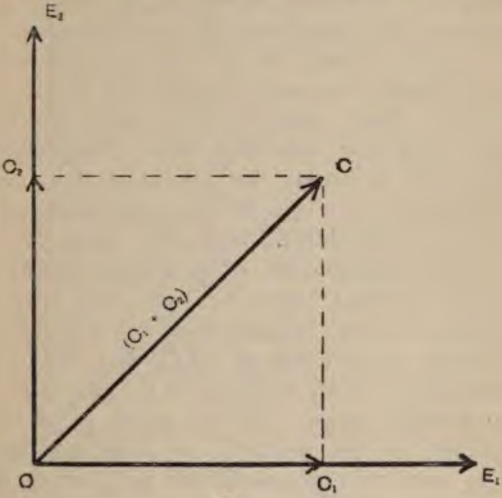

FIG. 70.

Again, on account of the symmetry of the arrangements, half of this component of the necessary total E.M.F. may be considered as being provided by E_1, and half by E_2. We must, therefore bisect $O\,a_3$ at m, and complete the parallelograms $O\ m\ E_1\ a_1$ and $O\ m\ E_2\ a_2$, which will give us the E.M.F. components $\overline{O\,a_1}$ and $\overline{O\,a_2}$. These are evidently the components of the total E.M.F.'s, E_1 and E_2, which are necessary to overcome the resistance, r, of the outer conductor and R of the load, and send the two separate currents through this portion of the circuit before they unite to flow back through the common conductor (see Fig. 69). The currents C_1 and C_2 must, of course, be drawn in phase with a_1 and a_2 respectively. As to the magnitude of these currents, this will depend upon the value of $r + R$; but, on the assumption of a given *power* to be transmitted — equal, let us say, to the power transmitted in the arrangement to which Fig. 70 applies — the projections of $\overline{O\,C_1}$ and $\overline{O\,C_2}$ on the corresponding E.M.F. vectors, $\overline{O\,E_1}$ and $O\,E_2$, must have a definite value. If the point p in Fig. 71 corresponds to the point C_2 of Fig. 70, we have merely to erect the perpendicular $p\ C_2$ to $O\ E_2$, and the point C_2, where it meets $O\ a_2$, gives us the vector $\overline{O\,C_2}$, representing the current in the outer conductors corresponding to the particular amount of *power* which each phase has to transmit. (The vector $\overline{O\,C_1}$ is obtained in a similar manner, being equal to $O\ C_2$, in the case of a balanced load such as we are considering.)

The total current $(C_1 + C_2)$ is obtained by compounding C_1 and C_2, and is represented by the vector and we now have all the necessary particulars le us to calculate the required section (or resistance) of the lines for a given loss of power in transmission. (It should not be necessary to point out that the resistance, r', of the common conductor $= \frac{O\,a_3}{O\,C}$, while $(r + R) = \frac{O\,a_1}{O\,C_1}\Big)$.

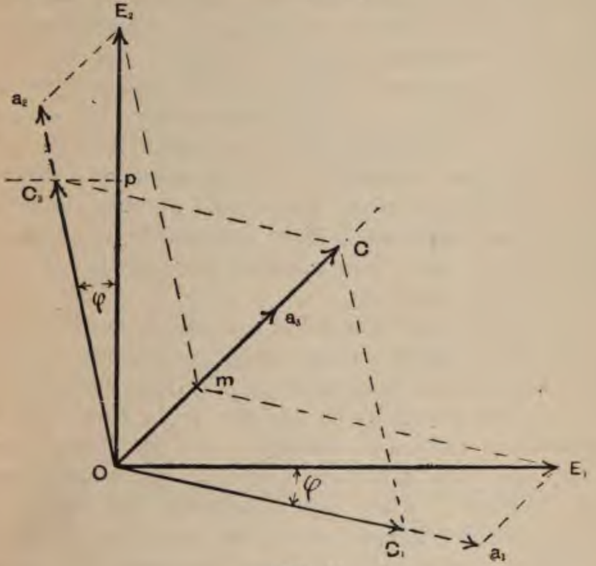

FIG. 71.

The important conclusion to be drawn from the diagram Fig. 71 is that the arrangement of a two-phase transmission line with a common return conductor, although it effects a certain saving in the necessary weight of copper, leads to *an increase of the phase angle between the two currents.* It is true that the displacement is somewhat exaggerated in the diagram, which is based on the assumption of an abnormally large resistance drop, and, in actual practice, the angles ϕ—with a reasonable $C^2 r$ loss, and the same current density in the three wires—would probably never exceed 5 deg.; that is to say, the phase difference between the currents, at times of full load, might be 100 deg. instead of 90 deg. As a matter of fact, long-distance transmission by two-phase currents is usually carried out by four wires, partly because of the above objection to the common conductor, but also because there are certain advantages in keeping the two phases entirely distinct. We may, therefore, save ourselves the trouble of constructing further diagrams, on the lines of Fig. 71, but very much more intricate in appearance, which would take into account the effects of self-induction and capacity.

Everything that has been said in connection with single-phase transmission being applicable to the case of two-phase transmission with four wires, there is very little more to be added. It is true that, in addition to the self-induction of the lines, each circuit will have a certain inductive effect upon the other, especially when (as is usually the case) the two circuits are supported by the same set of poles. But with a reasonable distance between the wires of the two circuits, the effects of mutual induction, even on a long line, are negligible. If necessary, it would be easy to cross the wires of one of the circuits at the middle point of the line, which would practically eliminate these effects.

(64) **Self-Induction of Two-Phase Line with Common Return.**—As the calculation of the *self-induction* of a two-phase line with common return may present some difficulties, it is proposed to deal

briefly with this question before passing on to the subject of three-phase transmission.

In order to simplify the diagram, we shall make the assumption that the middle conductor is without resistance, but that the whole of the resistance of the line is divided equally between the two outer conductors.

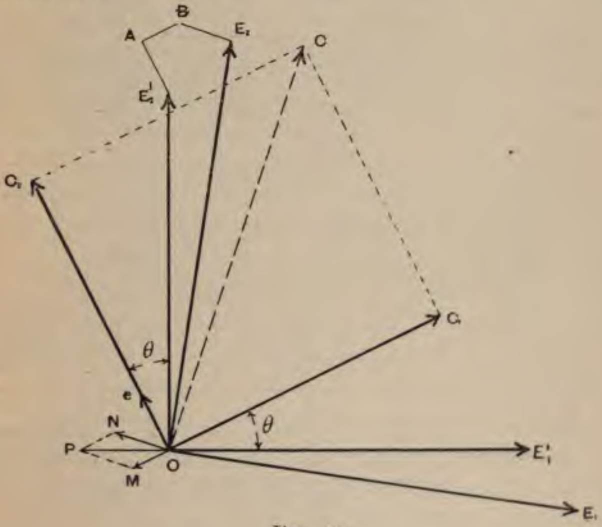

FIG. 72.

In Fig. 72, $\overline{O\ E'_1}$ and $\overline{O\ E'_2}$ are the pressures at the distant or receiving end of the line: there is an angle of 90 deg. between these vectors, to represent the phase difference of a quarter period. The two currents, C_1 and C_2, which will flow in the outer wires are shown lagging behind their respective pressures by an angle, θ, depending upon the power factor of the load. The dotted vector, $\overline{O\ C}$, gives us the magnitude and phase of the resultant current in the common middle conductor, of which the resistance is supposed to be negligible. The drop of pressure due to the resistance of the outer conductor conveying the current C_2 will be $\overline{O\ e}$; but, as regards the drop due to self-induction, this may be considered as the resultant of two E.M.F.'s, the

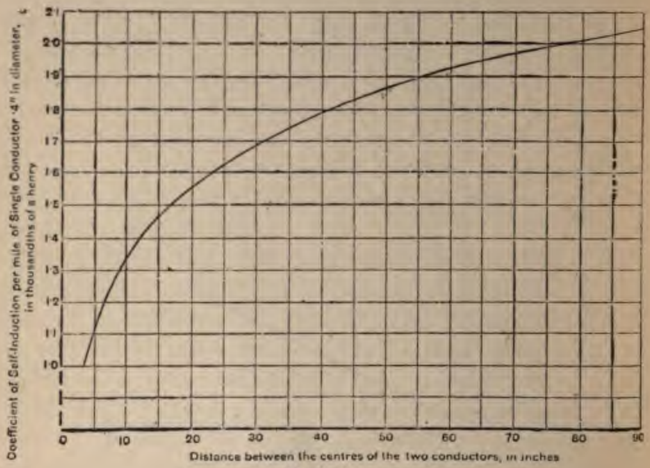

FIG. 61.

first, $\overline{O\ M}$, calculated on the assumption that the total magnetism in the loop is due to C_2, and the second, $\overline{O\ N}$, calculated on the assumption that the total magnetism in the loop is due to the current, C, in the common conductor. Each component E.M.F. must evidently be drawn 90 deg. behind the current to which it owes its origin, and by compounding these we get $\overline{O\ P}$ as the total E.M.F. of self-induction. We are now in a position to draw the vector $\overline{O\ E_2}$, representing the pressure at the generating end of the line. It is obtained by adding to $\overline{O\ E'_2}$ the three forces—$\overline{E'_2\ A}$, equal to $\overline{O\ e}$; $\overline{A\ B}$, equal but opposite to $\overline{O\ M}$; and $\overline{B\ E_2}$, equal but opposite to $\overline{O\ N}$. The vector, $\overline{O\ E_1}$, for the other phase is obtained in a similar manner.

With regard to the actual calculation of the lengths O M and O N, it will be remembered that the curve Fig. 61, reproduced on preceding page for convenience, gives us the coefficient of self-induction per mile of *single conductor*. The advantage of this will now be apparent, as we have merely to calculate the E.M.F.'s on the supposition that each wire forming the loop is producing a magnetic field independently of the other; bearing in mind that the E.M.F. component $\overline{O\ M}$ is due to the current, C_2, in one of the wires, while $\overline{O\ N}$ is due to a current equal to $C_2 \times \sqrt{2}$ in the other wire; these two E.M.F.'s being compounded in the usual way as indicated in Fig. 72.

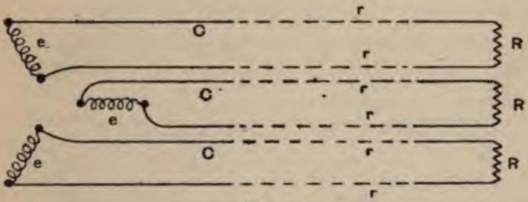

FIG. 73.

(65) **Transmission by Three-Phase Alternating Currents.**—We shall again assume, in the first instance, that the line is without self-induction or capacity. If we were to run six separate conductors—*i.e.*, two for each phase—we should merely be

transmitting three independent single-phase currents, the arrangement being as shown in Fig. 73; and if

e = the potential difference at the terminals of each circuit;

C = the current in each wire;

r = the resistance of each wire;

R = the resistance of the "load" on each phase,

then the total power transmitted would be

$$W = 3\,(e \times C),$$

or,

$$W = 3 \times C^2 \times (R + 2\,r)$$

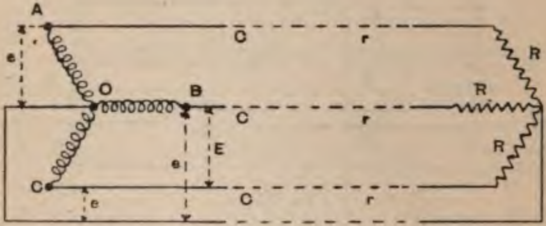

FIG. 74.

The pressure lost in transmission would be $2\,r \times C$, and the total power lost in the lines would be $3 \times C^2 \times 2\,r$.

Let us now see what is the effect of providing a common return for these three circuits. The arrangement is shown in Fig. 74.

The pressure at generating end between the three terminals of the alternator and the common return, or neutral point, is still e volts, and the total power transmitted is still $W = 3\,(e \times C)$; but, owing to the fact that the sum of the three outgoing currents is zero (seeing that they differ in phase by 120 deg., as shown in Fig. 75, and that any one current, such as $\overline{\text{O B}}$, is exactly equal and opposite to the resultant of the other two currents), there will be no current flowing in the common return conductor, and both pressure drop and $C^2 r$ losses in lines are, therefore, reduced to *one-half* of what they were with the arrangement of three separate circuits; the power loss in the lines being now $3 C^2 r$. This clearly shows how the transmission by three-phase currents is more economical as regards line losses than single-phase transmission. But it must not be overlooked that, in order to obtain a reduction by half

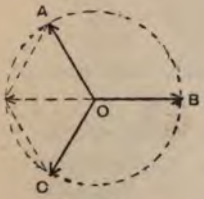

FIG. 75.

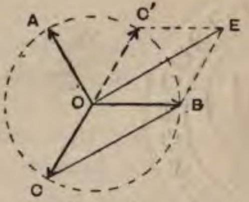

FIG. 76.

of the weight of copper in the lines, the pressure between wires is greater on the three-phase system than on a single-phase system transmitting the same power. Thus, the pressure, E (Fig. 74), between any two of the three transmission wires is equal to $e \times \sqrt{3}$, as shown by the diagram Fig. 76. Here the E.M.F.'s in the three sections of the alternator windings are represented by $\overline{\text{O A}}$, $\overline{\text{O B}}$, and $\overline{\text{O C}}$; and since the E.M.F., E, between any two terminals, such as B and C (Fig. 74) is the resultant of the E.M.F.'s acting in the two windings O B and O C connected in series, we must subtract one of these

($\overline{\text{O C}}$) from the other ($\overline{\text{O B}}$). Thus the resultant, $\overline{\text{O E}}$ (Fig. 76), is obtained by *adding* to the vector $\overline{\text{O B}}$ an imaginary vector, $\overline{\text{O C}'}$, exactly equal but opposite to $\overline{\text{O C}}$. This resultant is evidently equal and parallel to the line, C B, joining the ends of the two vectors $\overline{\text{O B}}$ and $\overline{\text{O C}}$, and it can be shown to be exactly $\sqrt{3}$ times greater than either of these vectors. We may, therefore, write

$$E = 1{\cdot}732\ e.$$

The *power* of a three-phase circuit, which is equal to three times $e \times C$, can evidently also be written

$$W = 3\left(\frac{E}{\sqrt{3}} \times C\right)$$

or,
$$W = \sqrt{3}\ E \times C,$$

where E is the pressure between any two of the three wires.

In comparing three-phase with single-phase transmission *on the basis of the same maximum potential difference between wires*, we have, for the total three-phase power

$$W = \sqrt{3}\ E \times C,$$

which, for equal power single-phase, would be written

$$W = E \times (\sqrt{3}\ C).$$

Let r = the resistance of each conductor on the three-phase system, and

r_1 = the resistance of each conductor on the single-phase system,

then *weight of copper, single phase* $\propto 2 \times \frac{1}{r_1}$

and *weight of copper, three phase* $\propto 3 \times \frac{1}{r}$

and the ratio

$$\frac{\text{weight of copper, single phase}}{\text{weight of copper, three phase}}$$

may be written $\frac{2\,r}{3\,r_1}$

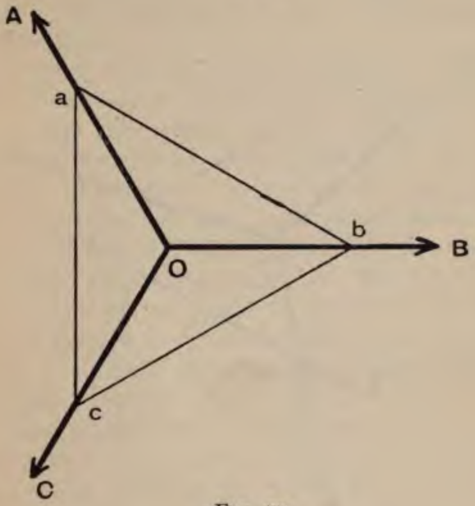

FIG. 77.

But for equal line losses we have

$$3\ C^2\,r = 2\,(\sqrt{3}\ C)^2\,r_1$$

which gives us $r = 2\ r_1$.

Hence the relative weights of copper are as 4 : 3, showing a saving of 25 per cent. in favour of the three-phase system.

(66) **Effect of Inductive load on Line Losses** We shall still neglect the capacity and self-induction

of the lines, but briefly consider the effect of an inductive load on the vector diagrams.

In Fig. 77, the diagram has been drawn for the condition of a power factor of unity.

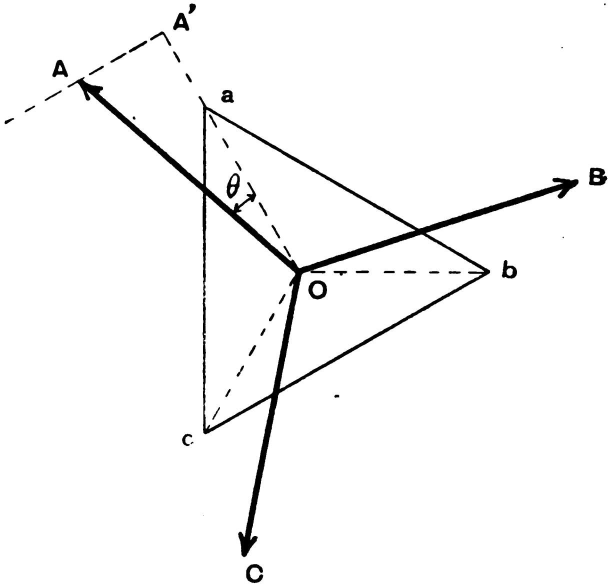

FIG. 78.

Here $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ are the three current vectors.

The potential difference between wires is

$$ab = bc = ca = \text{E}.$$

The length $\text{O}a = \text{O}b = \text{O}c = e = \dfrac{\text{E}}{\sqrt{3}}$

The total power is:

$$\begin{aligned} W &= \sqrt{3}\, E \times C \\ &= 3\, e \times C \\ &= 3\, (O\,A \times O\,a) \end{aligned}$$

In Fig. 78, the diagram has been drawn for an inductive load.

Here there is a certain displacement of the current phases relatively to the E.M.F. phases. It will be noticed that the vertices of the E.M.F. triangle no longer lie on the current lines as in the previous diagram. The three current vectors still make the same angle of 120 deg. with each other, but they have been moved bodily round (in the direction of retardation) through an angle θ. The total power is, evidently, no longer equal to three times $O\,A \times O\,a$, but to $3\,O\,A' \times O\,a$, where $O\,A'$ is the projection of $O\,A$ on $O\,a$; and $\cos\theta$ is the power factor of the three-phase circuit.

It was explained in connection with single-phase currents that, for a given total power to be transmitted at a definite pressure, the weight of the conductors—if the $C^2 r$ losses are to remain constant—must vary inversely as the square of the power factor. This rule necessarily applies equally to the case of three-phase transmission. Thus, in Fig. 78, the power factor ($\cos\theta$) is expressed by the ratio $\frac{O\,A'}{O\,A}$; but, if the *power* and *pressure* are constant, the length $O\,A'$ will not alter, whatever may be the power factor. Hence $O\,A$ (the current in any one of the wires) will vary inversely as the power factor, and—for a given loss in the line—the cross-section of the conductor must vary as the square of the current, *i.e.*, *inversely* as the square of the power factor.

It is not necessary to complicate the diagram Fig. 78 by drawing the vectors showing the effect of the resistance drop in displacing the E.M.F. phases at the receiving end relatively to those at the transmitting end. The construction is exceedingly simple, and can be carried out for each phase exactly as in the case of Fig. 59 (which is reproduced on this page). We have merely to bear in mind that O A and O *a* in Fig. 78 correspond with O C and O E′ in Fig. 59. The result will be an E.M.F. triangle representing the necessary pressures at generating end, which will be similar to, but somewhat larger than *a b c*, and of which the phase

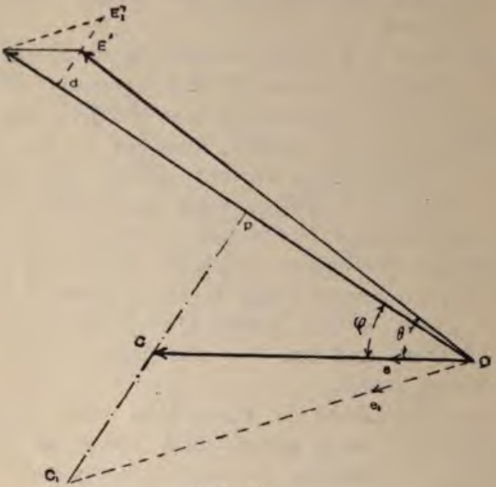

FIG. 59.

displacement angle (ϕ) relatively to the current vectors will be *smaller* than the angle, θ, at receiving end. In fact, the conclusions drawn from Fig. 59 as applying to single-phase transmission, will apply equally to the case of three-phase transmission; that is to say:

1. The power factor at generating end is greater than at receiving end.

2. *For a definite percentage* $C^2 r$ *loss in the conductors,* the arithmetical difference in the pressures at the two ends of the line will *diminish* as the load becomes more inductive.

It must not be forgotten that these conclusions refer to a line of negligible self-induction and capacity.

(67) **Self-Induction of Three-Phase Lines.**—The arrangement of three-phase overhead lines is generally such that the wires occupy the vertices of an equilateral triangle. Under such a condition, it is evident that the magnetic flux due to one of the wires will neither increase nor decrease the amount of the induction through the loop formed by the other two wires. As a matter of fact, if the three wires are arranged in any other practical manner, the effect of the induction due to any one wire on the loop formed by the other two wires will be found to be negligible.

It will be remembered that, when dealing with two-phase transmission with common return, it was shown how the total induction in any one loop may be considered as the resultant of two separate fields—one due to the current in the common return, and the other due to the current in the outer wire—combined at an angle of 45 deg.

In a three-phase circuit, the currents in the three wires are equal; but they differ in phase by 120 deg. If, therefore, we calculate—by the help of the curve Fig. 61—the induced E.M.F. on the assumption that each wire produces its own flux of magnetism

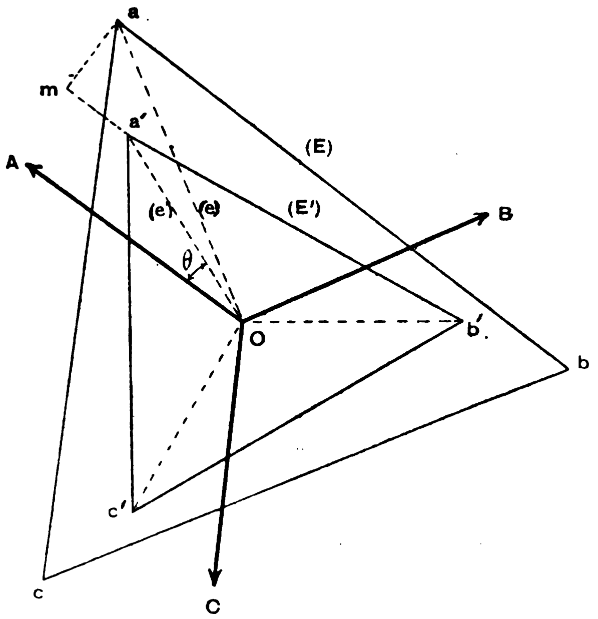

FIG. 79

independently of the other wires, we have merely to combine two such E.M.F.'s in the manner previously indicated (see Fig. 76, p. 199) to obtain the total E.M.F. in any one of the three loops. This is evidently $\sqrt{3}$ times as great as the induced E.M.F. calculated for the current in one of the wires only.

By far the simplest way of constructing the vector diagrams for three-phase circuits is to consider the three phases independently, and Fig. 79 shows the effect of taking into account the self-induction and ohmic resistance of the lines.

Here A, B, and C are the current vectors, and a' b' c' is the E.M.F. triangle for the receiving end; the angle of lag being θ.

Consider $\overline{\mathrm{O\ A}}$ and $\overline{\mathrm{O}\ a'}$ as being the current and E.M.F. vectors of a single-phase circuit. Draw $a'\ m$ parallel to O A to represent the ohmic drop due to the resistance of one conductor, and $m\ a$, at right angles to O A, to represent the necessary impressed voltage to balance the E.M.F. of self-induction, on the assumption that the magnetic field is due solely to the current, O A, *in one conductor*. Join O a, and note that, so far, the construction is exactly similar to that adopted in Fig. 60 for a single-phase circuit. We have merely to suppose the same construction to be followed in the case of the other two phases, and the result is the triangle $a\ b\ c$, which represents the necessary three-phase applied E.M.F. at the generating end.

The conclusions drawn from the diagram Fig. 60 when treating of single-phase currents, are equally applicable to the case of three-phase transmission.

Assuming the distance between the wires to be the same for a three-phase as for a single-phase transmission, let us see whether either system has an advantage over the other from the point of view of inductive drop.

In the first place, the total power delivered on the three-phase system (see diagram Fig. 79) is

$$\begin{aligned} &3\,\mathrm{O\,A} \times \mathrm{O}\,a' \times \cos\theta \\ &= 3\,\mathrm{C} \times e' \times \cos\theta \\ &= \sqrt{3}\,\mathrm{C} \times \mathrm{E}' \times \cos\theta \end{aligned}$$

where C = the current in any one conductor and E′ = the pressure between wires at receiving end.

In a single-phase system supplying the same power at the same pressure between wires, the current is, therefore, $\sqrt{3}$ times as great.

The inductive drop in one of the three-phase *loops* is proportional to C × $\sqrt{3}$, and exactly equal to the inductive drop due to each of the single-phase *wires*; hence, if the diameters of the wires were the same in both cases, the inductive drop in a three-phase line would be only *half* that in an equivalent single-phase line.

As a matter of fact, the diameter of the conductors in the single-phase line would be greater, on account of the larger current, and has the effect of *reducing* the self-induction on the single-phase line by something under 10 per cent.

(68) **Capacity in Three-Phase Transmission Lines.**—In their paper read before the American Institute of Electrical Engineers in 1900, Messrs. Perrine and Baum showed that the capacity of a three-phase line with the wires arranged in the form of an equilateral triangle may be considered as being made up of three equal capacities, all measured between any one wire and the line of zero potential, which may be taken as a line passing through the centre of the triangle. The three capacities are shown diagrammatically in Fig. 80.

The capacity of each of the three imaginary condensers shown in the diagram is given as

$$K = \frac{\cdot 0388}{\log_{10} \frac{d}{r}}$$

where K = the capacity *per mile* in microfarads ;
d = the distance between wires ;
r = the radius of any one wire.

It follows that the capacity current per conductor can readily be calculated exactly as explained in connection with single-phase transmission, the condenser E.M.F. being taken as the pressure between the wire and neutral line, which is equal to $\frac{1}{\sqrt{3}}$ times the pressure between wires.

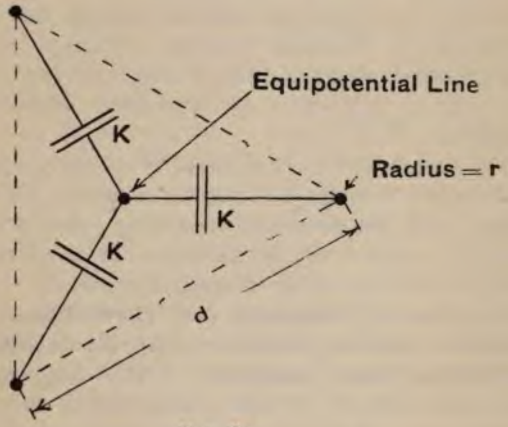

FIG. 80.

This method of calculating the capacity effects on a three-phase line is very simple and convenient ; it is evidently based on the assumption that the influence of the earth may be neglected; and this assumption is probably justified if the distance between the wires is small as compared with their height above ground.

The effects of capacity are not usually very great on overhead lines, but, in the case of long lines, transmission at light loads may become difficult, if not impossible, and if this should be the case, some means of neutralising the capacity may have to be adopted. Suitable choking coils might be connected between the wires at intervals along the line; or, if power is supplied through induction motors at intermediate stations, there may be no need for any special appliances to introduce additional self-induction.

The formulæ for the capacity of three-core cables with lead coverings are not so simple as for overhead wires, and information regarding the amount of capacity current to be expected in a given type of cable should be obtained from the maker.

(69) **Vector Diagram for Three-Phase Transmission, taking into account Resistance, Self-Induction, and Capacity.** — The diagram Fig. 81 has been drawn to scale, and shows the relations between the E.M.F.'s at generating and receiving ends of an overhead three-phase transmission line, for the condition of full load.

The data upon which the calculations have been based are such as might apply to a practical scheme of power transmission; and the diagram has the advantage of showing the relative importance of the effects due to ohmic resistance, self-induction, and capacity.

Data. — Total power delivered at full load = 4,000 kilowatts.

Power factor ($\cos\theta$) = ·8.

Pressure between wires at receiving end (E') = 30,000 volts.

Frequency (n) = 25.

Length of line = 100 miles.

Diameter of copper conductors = ·4 in.

Distance between centres of wires = 45 in.

To calculate the current, C', at receiving end, we have:

$$\sqrt{3}\, E' \times C' \times \cos\theta = 4{,}000 \text{ kw.},$$

which gives us 96 amperes as the full-load current, C', delivered at receiving end.

We can, therefore, draw the E.M.F. triangle $e'\ a'\ b'$ with equal sides, each of such a length as to represent 30,000 volts; and the vector $\overline{O\ C'}$ 36° 52′ behind $\overline{O\ e'}$ (corresponding to the power factor of ·8) and of such a length as to represent the current of 96 amperes at receiving end.

Capacity Current.—On the assumption that the whole of the capacity is concentrated at receiving end, and that the E.M.F. wave is a sine curve, we can calculate the capacity current by means of the formula

$$C_k = 2\,\pi\, n\ K\ e' \div 1{,}000{,}000,$$

where K is the calculated capacity between any one wire and the neutral line (as previously explained) and e' is equal to $\dfrac{E'}{\sqrt{3}}$.

The value of K, as given by the expression

$$K = \frac{\cdot 0388}{\log_{10} \dfrac{d}{r}} \times 100$$

works out at 1·65 microfarads, which gives us for the capacity current

$$C_k = 4 \cdot 5 \text{ amperes.}$$

The vector $\overline{O\,C_k}$, in the diagram, must therefore be drawn of such a length as to represent 4·5 amperes to the same scale as $\overline{O\,C'}$, and in a direction at right angles to O e', as explained in connection with single-phase currents. The total current in the line is $\overline{O\,C}$, obtained by compounding $\overline{O\,C'}$ and $\overline{O\,C_k}$. This vector, which scales 93 amperes, is, therefore, the current delivered to the line at the generating end.

Resistance Drop.—The resistance of a copper wire ·4 in. diameter and 100 miles long will be about 35 ohms. This gives us, for the drop in volts per conductor at full load, $C\,r = 3{,}250$ volts.

Self Induction.—On referring to the curve Fig. 61, it will be found that the coefficient of self-induction per mile of single conductor ·4 in. diameter, with a distance of 45 in. between wires, is 1·83 millihenrys, which gives us ·183 henrys for the line 100 miles long.

Hence the E.M.F. of self-induction per conductor (on the assumed condition of a sine curve current wave) is

$$\begin{aligned} & 2\,\pi\,n\,L\,C \\ &= 2\,\pi \times 25 \times \cdot 183 \times 93 \\ &= 2{,}670 \text{ volts.} \end{aligned}$$

We are now in a position to complete the diagram Fig. 81. Draw $e'\,m$ parallel to O C and of such a length as to represent the volts required to overcome the ohmic resistance of the conductor—*i.e.*, 3,250 volts. At the point *m* erect the perpendicular *m e* to represent the impressed volts (2,670) necessary to balance the E.M.F. of self-induction, and join O *e*. The length of this line

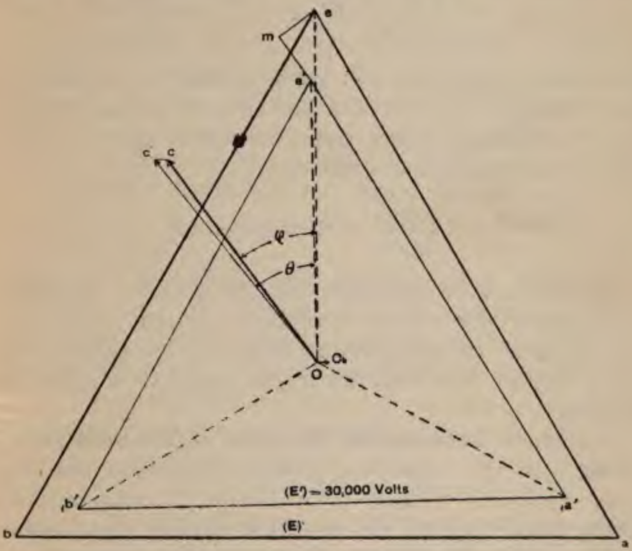

FIG. 81.

will evidently give us the value of $\frac{\text{E}}{\sqrt{3}}$, where E stands for the pressure between wires at the generating end. The sides of the large triangle scale nearly 37,500 volts, and this is the necessary pressure at generating end. The power factor at generating end (cos ϕ) is, in this example, very nearly the same as at receiving end; it measures tage. If the cables are insulated, and the economical current density is so high as to raise the temperature unduly, two or more cables would have to be laid side by side and connected in parallel, or the cross-section of the single cable would have to be increased, according to which method proved to be the cheapest.

Prof. George Forbes once stated* that he would like to see an accurate volume published on the correct use and development of Kelvin's law of economy. The writer has no such desire; neither can he appreciate the utility of a book of this kind. It would, in all probability, resolve itself into a compilation of more or less intricate formulæ, applicable to a number of imaginary cases which might, or might not, fit in with the conditions to be dealt with in practice.

The following graphical method of determining the most economical section of conductors, which was introduced some time ago by Mr. Edw. W. Cowan and the writer,† is applicable to any system or conditions of supply, and does not involve a knowledge of higher mathematics.

As already stated, in calculating the most economical section of the conductors in a transmission line, there are two factors only to be considered:

1. The first cost of the conductors.
2. The cost of the energy lost in the line.

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* "On Distant Electric Power Transmission," *Journal*, Inst. E. E., vol. xxix., May, 1900.

† "The Regulation of Pressure, and the Reduction of Light Load Losses in Alternating-Current Systems of Electric Supply." Paper read before the Northern Society of Electrical Engineers, March 23, 1896.

Kelvin's law, in its simplicity (and, indeed, when the law of maximum economy departs from this simplicity, it ceases to be Kelvin's law), may be stated as follows: "The most economical section of a feeder is that which makes the annual cost of the $C^2 r$ losses equal to the annual interest on the capital cost of the copper in the line, plus the necessary annual allowance for depreciation." The cross-section should, therefore, be determined solely by the current which the conductor has to carry, and not by the length of the line, or an arbitrary limit of the percentage full load drop in pressure; the drop, even if considerable, must take care of itself. If there are reasons which make a large drop undesirable, then, if necessary, economy must be sacrificed, and the line calculated on the basis of pressure drop only.

Kelvin's law is based on the assumption that bare conductors are used and that the cost of these, erected in position, is directly proportional to the weight of the copper. This assumption is not justified in practice if insulated cables are used, the cost of which, per pound of copper, is greater for the small than for the larger sizes. But, in the case of an overhead transmission line with bare conductors, Kelvin's law may give sufficiently accurate results, because, although the cost of erecting the smaller wires may be greater *per pound* than for the larger sizes, the insulators, supports, etc., may become more costly as heavier wires are used, thus making that portion of the total cost of the line *which depends upon the section of the conductor* approximately proportional to the weight of same.

To cover all cases in practice, the law of maximum economy may be stated as follows: *The annual cost of the energy wasted per mile of the transmission line, added to the annual allowance (per mile) for depreciation and interest on first cost, shall be a minimum.*

It is not very easy to estimate with any degree of accuracy the probable amount of the $C^2 r$ losses during the year's working. The energy lost in a given conductor depends upon the square of the current and the time during which the current is flowing. If a cable conveys a current C only twelve hours out of the twenty-four hours, then the energy wasted per day is only half what it would have been had the current been flowing continuously, and the watt-hours may be expressed as $12 \times C^2 \times r$, or as $24 \times \frac{C^2}{2} \times r$: it follows that, in working out the energy lost per annum in the conductor, we have to multiply the resistance, not by the square of the maximum current which the conductor will at times be carrying, but by the mean value of the square of the current throughout the year. The expected average daily load curve should therefore be drawn, and the mean value of C^2 calculated therefrom. If the capacity current is considerable, this must be taken into account when calculating the average value of the square of the current. The diagram Fig. 82 will serve to explain how the proper size of conductor may be worked out in actual practice.

Let us suppose that 10 per cent. is to be allowed for depreciation and interest on cost of conductor; then, in the diagram Fig. 82—where the horizontal distances represent resistance per mile, and the vertical distances represent money—plot the curve A, which gives the relation between 10 per cent. of the capital spent on the conductor, and its resistance in ohms per mile. Now calculate the cost of the $C^2\,r$ losses per mile of cable, for any particular value of

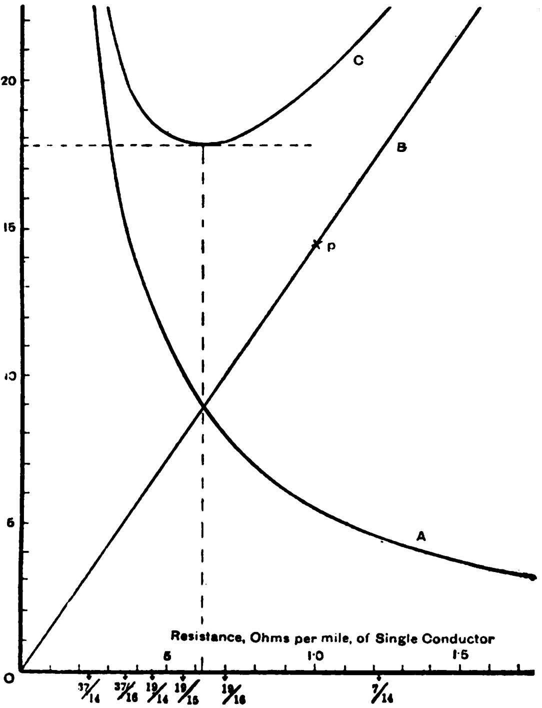

FIG. 82.

the resistance, and draw the straight line O B through the origin, which will give us the cost for any other resistance of conductor.

By adding the ordinates of the curves A and B, the curve C is obtained, of which the minimum value corresponds with the resistance per mile of the conductor which will be the most economical to use, whatever may be the length of the line or the pressure required at the receiving end.

Let us briefly consider how the two costs, of which the sum is to be a minimum, should be estimated.

In the first place, the capital outlay on which the interest is calculated must include the insulation on cables (if any) and the difference in cost (if any) of the conduits or overhead construction *which is dependent upon the size of the conductors.* No capital charge which is *constant* for all sizes of conductors within practical limits need be taken into account.

This is a point on which there appears to be some difference of opinion. It has been most emphatically stated, on the one hand, that all standing charges, such as trenching, erection of poles, etc., must be included; and, on the other hand, it has been asserted with equal assurance that these constant charges must, on no account, enter into the calculations of line efficiency. The truth of the matter is that it makes not the slightest difference, either one way or the other, whether or not these standing charges are included. Here the advantage of the graphical solution of such problems becomes apparent; because even the non-mathematical reader will readily understand that the effect of adding the same length to all the ordinates of the curve A (Fig. 82) will be to raise the curve C by the same amount, *without altering either its shape, or the position* (*in the horizontal direction*) of its point of minimum value.

With regard to what is a reasonable interest to allow on the cost of the conductors, for the purpose of drawing the curve A (Fig. 82) Prof. Forbes has pointed out* that, if the copper in the lines be mortgaged, the interest we have to consider is merely that paid on the mortgage—say 4 per cent. If the copper is paid for out of capital, the interest may be much greater, being equal to the estimated dividends to be paid on the capital invested. In addition to this there is the annual allowance to be made for depreciation, which will depend upon the probable life of the line, and its scrap value when replaced.

We now come to the second of the two cost items—namely, the cost per annum of the energy wasted in the line.

This will generally be the works cost of the power, and must include interest and depreciation on the plant in generating station, including step-up transformers, controlling gear, boosters, etc., through which the current passes before reaching the line. At the same time, each scheme must be independently considered, as evidenced by the following statement borrowed from Prof. Forbes's paper previously referred to: When using water power which is unlimited compared with the demand, the price per annum of the horse-power is the cost of generating the power. When the water power is limited, and the demand unlimited, the price per annum of the horse-power is the selling price of the power.

There is also a correction to be made in the event of the economical losses in the line bearing a large proportion to the total power transmitted. In this case, the cost of power at the distant end of the line may be appreciably greater than at the generating end, and, under normal conditions, we ought to take the cost of the wasted power at something between these two values.

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* "On Distant Electric Power Transmission," *Journal*, Inst. E.E., vol. xxix., May, 1900.

If we determine the slope of the line O B in Fig. 82 on the basis of works cost at generating end, this will enable us to calculate the approximate resistance of the line and ascertain the approximate loss of power, together with increased cost per horse-power, at the receiving end. We can then revise our previous figure for cost of power wasted in line and obtain a corrected value of the required resistance.

Examples illustrating use of Diagrams.—Consider an overhead line with bare copper conductors, to carry a maximum current of 60 amperes. Let us assume that probable load curves have been drawn, which will enable us to estimate the $\sqrt{\text{mean square}}$ value of the current throughout the year. This might be about one-third of the maximum value; let us say 20 amperes. If, therefore, we multiply the resistance of each conductor by the square of 20, we shall obtain the total power lost in the line on the assumption that this loss is going on day and night throughout the year. This gives us as the total number of Board of Trade units wasted in each conductor per annum *for an assumed resistance of 1 ohm*:

$$\text{Units} = \frac{1 \times (20)^2 \times 24 \times 365}{1,000}$$

$$= 3,500$$

and if the cost of this power be taken at 1d. per unit, the cost would be

$$\frac{3,500}{12 \times 20} = £14{\cdot}6.$$

This enables us to plot the point p on the curve O B, and, by joining O p, we can read off the cost of power per annum for any other resistance of conductor.

With regard to the curve A, we shall suppose that 10 per cent. is to be allowed for depreciation and interest on capital spent on conductors; and, if we take the cost of these at 1s. 3d. per pound, we can readily plot a number of points through which this curve must be drawn. The curve C, as already explained, is the result of adding the ordinates of curves A and B. Its minimum value, in our example, corresponds to a resistance of ·62 ohms per mile, which gives us as the most economical conductor a stranded cable somewhere between $^{19}/_{16}$ and $^{19}/_{15}$, or the equivalent solid wire. (It should be noted that—since the curve A is a rectangular hyperbola—the minimum value of C is on the same ordinate as the point where the two curves cross—*i.e.*, where A is equal to B; but this is not necessarily the case when the conductors are provided with an insulating covering.)

In order to carry the numerical example a little further, we shall assume that the transmission is on the three-phase system 50 miles long, with 20,000 volts between wires.

The drop in volts per conductor will be

$$\begin{aligned} C \times r &= 60 \times {\cdot}62 \times 50 \\ &= 1{,}860 \text{ volts,} \end{aligned}$$

and if we disregard the effects of self-induction and capacity, the loss of pressure between wires at the receiving end will be

$$1{,}860 \times \sqrt{3} = 3{,}220 \text{ volts.}$$

The initial pressure being 20,000 volts,
The pressure at distant end is 16,780 volts.
The full-load power at generating end = 2,080 kw.
The full-load power at receiving end = 1,745 kw.
The power lost in the line = 335 kw.
And the efficiency of line = 84 per cent.

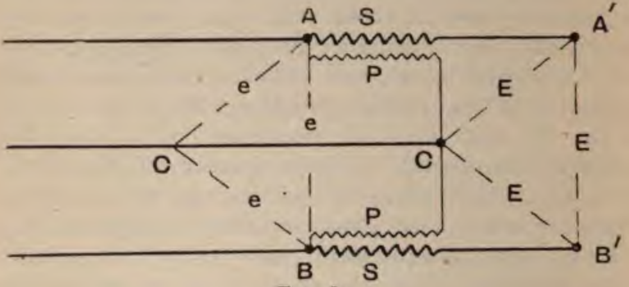

FIG. 83.

(71) **Regulation of Pressure at Receiving End.**—When the voltage drop is considerable, as will nearly always be the case in long transmission lines, it is necessary to raise it—*i.e.*, to regulate it so as to avoid variations with changing load.

This may be done on a Δ-connected three-phase system by means of two regulators only, as indicated in Fig. 83.

Here the points A, B, and C are the terminals of the transmission line. Imagine P and S to represent respectively the primary and secondary turns of a transformer. If we connect two such transformers in the manner indicated on the diagram, the effect will be to raise the pressure between *all three wires* to the extent of the volts added by the secondary turns, S.

Thus, if the pressure e = 16,000 volts, and the ratio $P : S = 4 : 1$, the pressures, E, will be 16,000 + 4,000 = 20,000 volts.

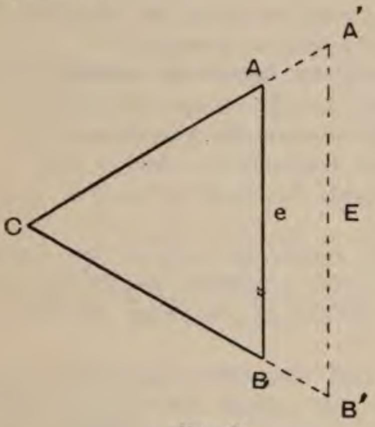

FIG. 84.

The diagram Fig. 84 will make this quite clear. Here $\overline{\text{A B}}$, $\overline{\text{B C}}$, and $\overline{\text{C D}}$ are the three vectors representing the pressures, e, before boosting up: $\overline{\text{A A}'}$ and $\overline{\text{B B}'}$ represent the added volts between A and A′ or B and B′ (Fig. 83). These added volts are evidently in phase with the pressures $\overline{\text{C A}}$ and $\overline{\text{B C}}$ respectively, because the potential difference at the secondary terminals of a well-designed transformer is always in phase with the primary E.M.F. We have only to complete the triangle C A′ B′ to see that the two transformers, connected up in the manner described, will do all that is required in the way of raising the pressure on a three-phase circuit.

It will be understood that regulation of this sort is usually carried out on the secondary side of the step-down transformers, but the principle is the same. Moreover, the transformers shown in the diagram Fig. 83 must be of the variable ratio type, so that the added volts may be varied (by hand or otherwise) to suit the alterations in the load.

(72) **Boosters at Intervals along the Transmission Line.**—In the event of a large pressure drop occurring between the generating and receiving ends of a long transmission line, it may be advantageous to provide boosters at one or more points along the line.

There would appear to be a certain vagueness in the minds of some engineers as to the real object and possible advantage of using boosters along a transmission line.

It is true that the energy put into the line at intermediate points cannot be cheaper—and, indeed, is usually more costly—than the energy supplied to the line at the generating end; but the booster system enables us to *keep up the pressure all along the line*, and hence the economy; provided always that the losses in the boosters themselves, their maintenance, and the necessary allowances for interest and depreciation do not counterbalance the saving.

As an example, consider a single-phase line conveying a current of 100 amperes at an initial pressure of 20,000 volts. Suppose the drop of pressure in the line to be 10,000 volts: this will leave only 10,000 volts at the receiving end.

The power put into line at generating end = 2,000 kw
The loss in the line = 1,000 kw.
The power available at receiving end = 1,000 kw.
Hence, *efficiency of line* = 50 per cent.

Now imagine a booster to be inserted at a point half-way along the line. This booster may be considered as a highly insulated alternator of 500 kw. capacity, capable of generating 100 amperes at 5,000 volts. The arrangement now becomes as shown in Fig. 85. The drop in the first section of the line is, as before, 5,000 volts; and the drop in the second section is evidently similar—*i.e.*, 5,000 volts—leaving the total waste of power in the line the same as before the booster was inserted. But by providing this booster at the middle point of the line, we have been enabled to raise the pressure here up to the maximum initial value of 20,000 volts, with the result that we now have 15,000 volts (or 1,500 kw.) available at the receiving end. We have certainly not succeeded in obtaining this increased power without having to pay for it; but note that, by keeping up the pressure, we have been able to obtain it *without increasing the losses in the line.*

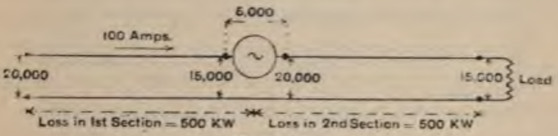

FIG. 85.

If, for the sake of simplicity, we neglect the question of booster efficiency, we now have:

Power supplied to the line = 2,000 + 500 = 2,500 kw.

Power lost in the line = 1,000 kw.

Power available at receiving end = 1,500 kw.

And for the *efficiency of the line* $\frac{1,500}{2,500}$ = 60 per cent.

Boosters are generally arranged so as to take their power from the generating end—that is to say, they take the form of transformers which provide the extra volts at the cost of a corresponding loss of current. A three-phase transmission line, in two sections, with booster at the end of the first section, is shown diagrammatically in Fig. 86.

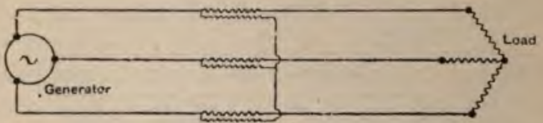

FIG. 86.

Such boosters must be of the variable ratio type, with hand or automatic regulation.

(73) **Relative Advantages of Different Systems of Transmission.** — By way of introduction, it should be mentioned that a comparison of various systems on general lines is not only useless, but impossible: there is no basis of comparison applicable to all conditions of practice, and each particular case must be considered on its own merits.

The following points should not be overlooked:

(1) In comparing the relative cost of conductors necessary for two different systems of transmission, the *efficiency* must be the same in both cases—*i.e.*,

the same amount of energy must be considered as being transmitted the *same distance* with the *same loss*.

(2) What is to be understood by the *pressure* must be clearly defined. For instance, when comparing continuous with alternating current transmission, it must not be forgotten that the stress on the insulation is less with continuous currents than with alternating currents at the same $\sqrt{\text{mean square}}$ pressure. In fact, on the assumption of a sine curve wave, it is easy to see that—for the same power transmitted, at the same efficiency, and *with the same maximum stress on insulation*—the weight of copper for a single-phase alternating-current transmission would be *double* that required if continuous currents were used.

Again, when considering alternating-current systems, it is necessary to decide whether it is the maximum potential above earth or the maximum pressure between wires which should be taken as a basis of comparison.

Assuming the middle, or neutral point, of the various systems to be "earthed," and allowing for a definite maximum pressure between any wire and earth, the comparison between the different alternating-current systems becomes a simple matter.

Let e and C stand respectively for the voltage and current *per leg* in either of the three systems shown diagrammatically in Fig. 87. Then, the total power transmitted in each case will be

$$e \, \mathrm{C} \cos \theta \times n,$$

where $\cos \theta$, as usual, is the power factor, and $n =$ the number of legs.

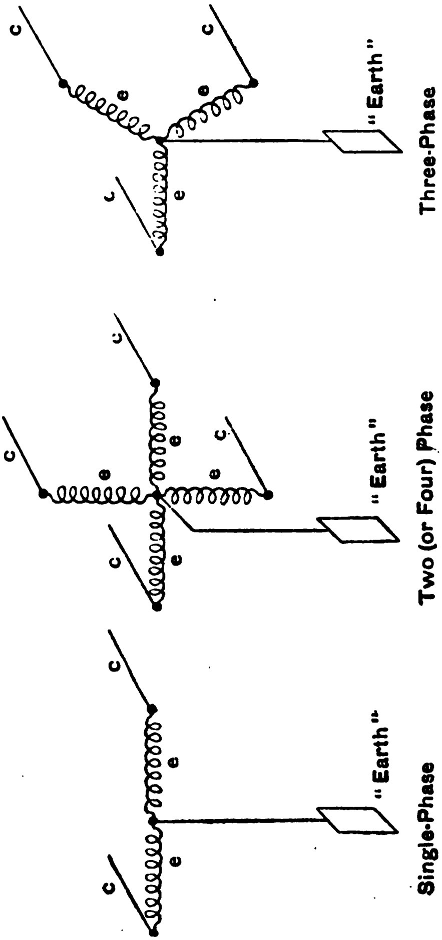
c
e
c
e
"Earth"
Single-Phase
c
e
c
e
c
e
c
e
"Earth"
Two (or Four) Phase
c
e
c
e
c
e
"Earth"
Three-Phase

FIG. 87.

If r be the resistance of each line conductor, the total line loss will be

$$C^2 \, r \times n,$$

and, for the same line efficiency, the weight of copper per horse-power transmitted will evidently be the same in all cases.

This leads us to the conclusion that, *for any polyphase system*, the power lost in the line depends only upon the joint resistance of the conductors, the power transmitted, and the power factor, *provided the pressure between any wire and the neutral point is constant.*

Let us neglect the capacity and self-induction of the line, and assume that $\cos\theta$ is the power factor of the load at receiving end. Then

$$\textit{power delivered} = n\, e\, C \cos\theta = W$$

and the *power lost in line* $= n\, C^2\, r$

This last quantity, expressed as a percentage of the power delivered, becomes

$$\textit{percentage power lost in transmission} = \frac{n\, C^2\, r \times 100}{W}$$

Let us denote by R_p the joint resistance of all the line conductors in parallel—*i.e.* $R_p = \frac{r}{n}$. This gives us, for the numerator of the above percentage loss, the expression

$$n^2\, C^2\, R_p \times 100,$$

But $n^2\, C^2$ is equal to $\frac{W^2}{e^2\,(\cos\theta)^2}$

Hence we may write:

$$\left.\begin{array}{l}\text{Percentage power lost in trans-}\\ \quad\text{mission for } \textit{any} \text{ polyphase}\\ \quad\text{system}\end{array}\right\} = \frac{W \times R_p}{e^2\,(\cos\theta)^2} \times 100$$

It might at first sight appear as if there was nothing to choose between the various systems shown in Fig. 87; but it must not be forgotten that the potential difference *between the wires* has not been taken into consideration. When this is done, the advantage will clearly be seen to be in favour of the three-phase system; thus, the maximum pressure between any two of the line wires will be 1·73 *e* for the three-phase system, and 2 *e* for the single or four phase systems.

Although we have assumed the case of an earthed neutral, there is much to be said in favour of a system without an earthed point, mainly as regards reduced risk of stoppage through a breakdown of insulation.

Where current for power and lighting is to be taken off the same mains, the two-phase system, with the phases kept entirely distinct, appears to offer some advantages; or the power may be transmitted by three-phase currents and converted into two-phase by a suitable arrangement of transformers, as first introduced by Mr. C. F. Scott. In this case the frequency must not be lower than, say, 30 for incandescent lighting or, say, 45 for arc lighting.

If the load consists of small motors and incandescent lamps, a three-phase distribution at a frequency of about 30 will give very fair pressure regulation if the secondaries of the transformers are Δ connected. Even with a difference in load of 25 per cent. between the phases, the alteration in voltage between the wires can be shown to be hardly noticeable, and the motor load will always tend to reduce the effects of any differences in the lamp loads.

Another method, applicable to a supply combining power and lighting, is to connect all the lights across one phase only of the three-phase supply, and keep the pressure steady on this particular phase, leaving the pressure across the other two phases to adjust itself according to the load. This should not interfere with the running of motors connected across all three phases.

If the power is to be used for electric traction, the frequency must be low, preferably about 15 periods per second. The two-phase system has been advocated for electric traction on the ground that—for a given power—the current in the (two) trolley wires will be less, while the current in the rails will be greater than with the three-phase system; thus effecting economy in the line. (The section of the rails is usually such that their conductivity is higher than that of the trolley wires.)

On the Valtellina Railway, the transmission from the water-power station is by three-phase currents at 20,000 volts; this is transformed down in sub-stations to 3,000 volts, a lower pressure being, in this case, inadmissible, as it would involve the collection of abnormally large currents from the trolley wires.

For transmission of power to a distance, the three-phase system would, on the whole, appear to have certain points in its favour.

(74) **Arrangement of Overhead Lines in Practice.**—It is not intended to deal with any questions concerning the practical construction of transmission lines, or the calculation of stresses and strains in wires or supports; but this chapter would not be complete without a reference to various minor

matters affecting the design of transmission lines from an electrical standpoint.

Distance between Wires.—The spacing of the wires will depend mainly upon the pressure between them. The formula

$$d = 6 + 1\tfrac{1}{4}\ V,$$

where $d =$ distance between wires in inches, and $V =$ pressure in thousands of volts, will be found to agree fairly well with the average of recent practice.

Height above Ground.—This will depend largely upon whether wooden or iron poles are used. There are many advantages in having high-tension wires a considerable distance above ground, and the practice of using iron poles or light towers spaced twelve or fifteen to the mile, in place of wooden poles spaced, say, forty to the mile, is becoming more and more common. Apart from the fact that the iron poles are more durable, the longer spans are an improvement from the point of view of insulation. Moreover, since fewer insulators are necessary, a higher price can be paid for them, and the best possible type adopted. In no case should any but the best and strongest insulators be used, as no ultimate saving will be effected through the use of cheap insulators.

The height of the wires above ground, when iron poles are used, might be from 30 ft. to 40 ft., or even more in cases where very long spans can be adopted with advantage.

Aluminium Wires. — These are largely used in America for overhead transmission. For the same conductivity as copper, their diameter will be about one and a quarter times as great and their weight almost exactly half as great as that of copper: the tensile strength of the aluminium conductor will be greater than that of a copper conductor of equal conductivity. The cost, in practice, works out about the same for either metal, or perhaps slightly in favour of aluminium.

The effects of snow, ice, and wind on the larger diameter of wires must not be overlooked. Again, since aluminium has a large coefficient of expansion, a greater sag must be allowed for than with copper. It has been noted that the sag in a span of aluminium wire 132 ft. long has been reduced from 33 in. at 100 deg. to 13 in. on a cold morning.

Aluminium is liable to be attacked and corroded in the neighbourhood of manufacturing centres; and trouble has also arisen through crystallisation, which sometimes results from continuous vibration. Stranded conductors are probably preferable to solid wires.

Bronze conductors have been recommended as being preferable to copper.

Protection from Lightning and Abnormal Pressure Rises.—This is a more important subject than can be adequately dealt with in these pages. The practice of running one or more earthed iron wires—whether ordinary or "barbed"—the full length of the line in the neighbourhood of the conductors, meets with little approval at the present day, probably owing to troubles caused by falling wires. A good type of lightning arrestor should be used at the stations, and along the line. The horn type is liable to cause short-circuits during rain and snow-falls, and should be covered. This type has also the disadvantage of being liable to set up electrical oscillations. The Wurtz arrestor, with resistance in series, or both in series and in parallel, would appear to be fairly satisfactory.

The question of protection from lightning and abnormal pressure rises is one which has not yet been satisfactorily solved. In many cases, static dischargers have been the cause of more trouble than they have prevented, and there is no doubt that it is advisable to reduce the number of safety devices as far as possible. At the same time, the most elaborate precautions may be found necessary to prevent interruption of supply during thunder storms.

The practice of insulating the beds and frames of alternators and transformers, which is fairly common in South Africa, would appear to have advantages; the principal disadvantage being increased danger to life.

The abnormal rises of pressure which are liable to occur when switching on or off a long transmission line may lead to trouble, in which case some device must be introduced with the switchgear to enable the pressure on the lines to be gradually raised or lowered. This can be done by inserting a variable-ratio hand-regulated transformer between the 'bus bars and the terminals of the line, this transformer being cut out of circuit after the switch has been closed on to the 'bus bars.

In the case of overhead conductors, it is not usual to insert any such charging devices; but if the line is a very long one, some method of gradually switching on and off may be necessary: where any considerable length of insulated underground cables is used, the sudden switching on or off of the pressure might lead to trouble.

Mutual Induction.—Although the question of self-induction in transmission lines has been treated at some length, no reference has yet been made to the inductive effects of one line on a second line running parallel with it, as would often occur in actual practice; the reason of this omission being that such inductive effects are generally negligible. In any case, they can be almost entirely avoided by suitably transposing or spiralling the lines.

Thus, if a three-phase line be arranged so that each wire occupies the central position for a distance equal to one-third of the entire length, it should have no appreciable inductive effect upon a neighbouring parallel conductor.

With two three-phase circuits supported on the same set of poles, it is usual to transpose the wires of one circuit in the opposite direction to those of the other; but spiralling should be avoided as far as possible.

The chief trouble is with telephone circuits, the wires for which are frequently carried by the same poles as are used to support the transmission lines. This is not good practice, if it can be avoided, seeing that the telephone will generally be needed most at times when there is trouble with the lines—*i.e.*, when possible out-of-balance currents in the overhead conductors are liable to interfere with conversation on the telephone. The wires for the telephone should, when possible, be supported on a separate set of posts. If suitable precautions are not taken, dangerously high voltages might easily be induced in the telephone circuit.

CHAPTER VII.

POLYPHASE INDUCTION MOTORS.

(75) The general principles underlying the rotation of a short-circuited armature in a rotary magnetic field, produced by two or more alternating currents differing in phase, were explained in article 28, Chapter III., to which the reader is referred, and with which he should make himself thoroughly familiar. A picture of the rotating field, dragging with it the short-circuited armature, is a very useful one; but there is another way of studying the behaviour of this class of motors which will generally be found to have advantages.

This consists in considering the induction motor as a slightly modified transformer, the primary coils of which are represented by the stator, or inducing windings, while the secondary coils are movable, being, in fact, the short-circuited windings of the rotor.*

It has been shown (see article 27, Chapter III.) how a rotary field may be produced by two or more currents having certain phase differences between them; and since two equal currents differing in phase by 90 deg. will produce a uniform rotating field, it follows that any uniform rotating field, however produced, can be resolved into two equal alternating components differing in phase by a quarter period; and, instead of studying the problems of induction motors by picturing a magnetic field revolving in space, we shall, in this chapter, treat the subject by imagining two alternating magnetic fields differing in phase by 90 deg., and acting together so as to bring about the rotation of the short-circuited armature.

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* This nomenclature is, of course, based on the assumption that the primary windings—*i.e.*, the coils connected to the supply mains—are at rest, while the short-circuited secondary revolves; but, although this is the most usual arrangement, it must be understood that it is by no means a necessary one. If the primary coils are wound on the revolving portion of the magnetic circuit, then a number of slip rings must be provided for conveying the current to the coils, corresponding in number to the number of phases of the polyphase supply.

This method is by no means new: indeed, it has always been advocated by Mr. Ll. B. Atkinson, and was used by him as long ago as 1898, in his classic paper on "Alternating-Current Motors," read before the Institution of Civil Engineers.*

Mr. Atkinson, however, treats the subject very fully and even goes into details of design; while the methods and diagrams adopted in the following pages, although not primarily intended for the use of designers, are yet sufficiently accurate for general purposes, and lead to considerable simplifications.

In Chapter V. the theory of the alternating-current transformer was explained, on the assumption that magnetic leakage was *nil*—*i.e.*, that all the magnetism generated in the primary circuit passed also through the secondary coils. But the alternating-current induction motor—owing to the disposition of the windings and the necessary air-gap—is by no means a perfect transformer, and we shall, therefore, consider first what are the effects of magnetic leakage in an ordinary single-phase transformer.

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* *Minutes of Proceedings*, Inst.C.E., vol. cxxxiii., p. 113.

(76) **Magnetic Leakage in Transformers.** — Unless the primary and secondary coils of a transformer are wound close together on the same portion of the iron core, there must necessarily be a certain number of magnetic lines generated by the primary current which do not thread their way through all the turns of the secondary coils. The amount of this leakage magnetism will increase with the growth of the primary current, and will, therefore, be approximately proportional to the value of the secondary current.

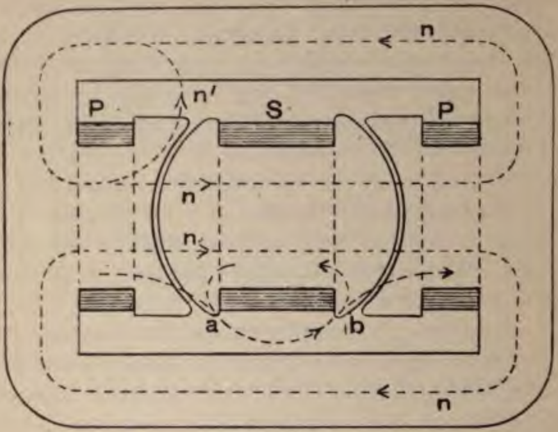

FIG. 88.

In Fig. 88 is shown a single-phase transformer in which the magnetic leakage would be considerable.

The two primary inducing coils, marked P, are wound upon the iron core at some distance from the secondary coil, S, and, moreover, an air-gap has been introduced in that portion of the magnetic circuit which lies between the two sets of coils.

When there is no current in the secondary—*i.e.*, when this is open circuited—nearly all the magnetic lines due to the primary current will pass through the central core, and so thread their way through the secondary coil, because—unless the air-gap be very large—this will be the easier path; and the amount of magnetism finding a shorter path, such as n', would be small. But when current is allowed to flow in the secondary coils, this produces a magnetising force exactly equal and opposite to the magnetising force of the main component of the current in the primary coil. The result is that, although this component of the primary current can produce no magnetic flux through the secondary coil, it will give rise to a considerable amount of magnetism which will leak across the air spaces and miss the secondary coils.

In treating of magnetic leakage in transformers and induction motors, the assumption is usually made that the secondary winding has self-induction; the argument being as follows:

The current in the primary coils gives rise to a total flux, N, of such a value as to choke back the whole of the applied potential difference which is not lost in ohmic drop.

Of this total flux, N_s lines pass through the secondary coils, and N_l lines leak back by other paths.

Now, the N_s lines through the secondary induce a certain back E.M.F. which, however, is not the effective E.M.F. in the secondary circuit, even if the terminals are short-circuited or the outside load is non-inductive. It is assumed that the secondary has self-induction, and that the current flowing in it gives rise to another E.M.F., in quadrature with the E.M.F. due to the flux N_s, thus reducing the effective E.M.F. producing the flow of current.

It is not suggested that this is an incorrect way of treating the subject (in some cases it may be a necessary one); but note that it involves the idea of a certain portion of the total leakage magnetism being *generated* by the current in the secondary coil; and we must, therefore, conceive of two *independent* streams of alternating magnetism (with a phase difference of a quarter period) passing through this coil. In other words, the magnetic flux due to the primary coil gives rise to a certain E.M.F. in the secondary coil; the current in the latter also produces a back E.M.F. of self-induction, which, together with the above E.M.F. of *mutual* induction, gives us the *resultant* or *effective* E.M.F. in the secondary winding.

The author's method, which leads to a certain simplification of the diagrams, is based on the assumption that the whole of the magnetic flux which passes through the secondary coil generates an E.M.F. in the secondary windings to which the secondary current is directly due, and that this is the *only E.M.F. generated in these windings*.

If, therefore, the external secondary circuit is without self-induction, the current in the secondary coils will be in phase with the induced E.M.F.; which is simply another way of stating that the secondary transformer coils are without self-induction.

Referring again to Fig. 88, it might be objected that the current in the secondary coil, S, is bound to produce leakage magnetism as represented by the dotted line *a b*; but, on consideration, it will be seen that any such magnetic lines may, with equal correctness, be considered as due to the component of the primary current which balances the secondary current; in which case the closed path of such lines will not be through the central iron core on which the secondary is wound, but through the outside portion of the magnetic circuit.

It is possible that the configuration of the magnetic circuit may be such as to render inaccurate this method of treating the subject; but, in almost any case which is likely to arise in practice, the error introduced by supposing the secondary to be non-inductive, will be negligible. The question practically resolves itself into the correct determination of the core losses in the various portions of the magnetic circuit.

(77) **The Induction Motor considered as a Special Case of the Alternate-Current Transformer.** — Fig. 89 represents a polyphase induction motor in its simplest form, and has been specially drawn to illustrate the resemblance between such a motor and an ordinary transformer with a large amount of leakage.

The revolving field is resolved into two equal components with a phase difference of 90 deg.; the two stator coils P P being connected to phase 1, while the two coils O O are connected to phase 2.

In practice, the motor would probably have more than two pairs of poles, and, moreover, the stator windings would be evenly distributed and wound in a number of slots on the inside periphery of the stator ring; but the arrangement shown in the figure is otherwise correct, and will serve the purpose of explanation better than a section showing the actual disposition of the windings in a two-phase motor.

The rotor windings may consist of three or more equally spaced separate short-circuited windings; or—if of the squirrel-cage type—all the conductors, passing through slots on the periphery of the rotor, would be short-circuited by a couple of rings at the ends of the armature.

In Fig. 89, each pair of diametrically opposed conductors is supposed to be short-circuited by end connectors, such as *n* and *m*.

The action of the motor may briefly be described as follows:

The coils P P of phase 1 produce an alternating magnetic field in the direction B B′. This does not induce any E.M.F. in the closed rotor circuit *n* which does not enclose any of the magnetic lines due to these exciting coils; but, in the other circuits, E.M.F.'s will be induced which will give rise to currents lagging exactly 90 deg. behind the magnetic flux passing through the rotor in the direction B B′; the maximum current being produced in the horizontal coil *m*.

If the motor is at rest, *no torque will be exerted by these currents reacting upon the field of phase 1*, and the motor would, therefore, not start with only the coils P P excited, even under light load conditions.

But note that the coils O O excited by phase 2 produce a field along the axis A A′ exactly 90 deg.

out of phase with the field B B′, and, therefore, exactly *in phase with the currents induced by phase 1 in the rotor windings.*

The result is that the magnetism entering the rotor from one set of poles, gives rise to currents in the rotor conductors in phase with the magnetic flux from the other set of poles; and since the torque will depend upon the product of the current and field strength—*i.e.*, upon the magnitude of the rotor currents and the intensity of that component

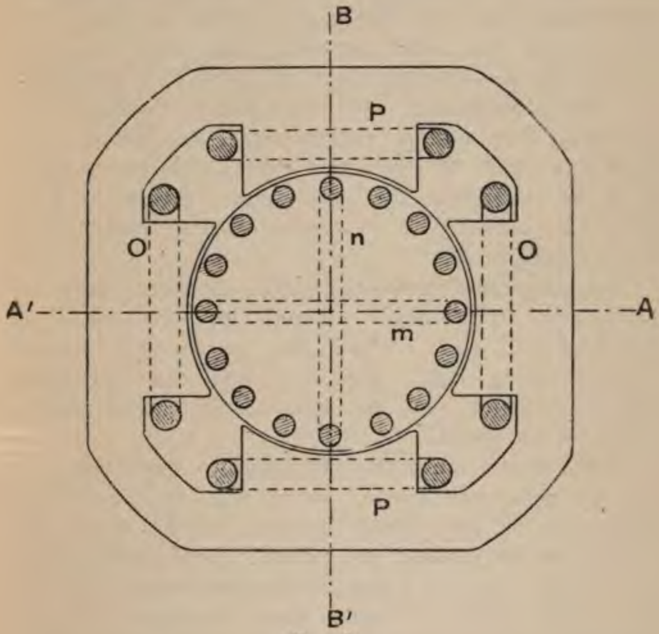

Fig. 89.

of the magnetic field in which the rotor conductors are immersed *which is in phase with the current*—it follows that the motor will be self-starting.

As it runs up to speed, the E.M.F.'s induced in the rotor conductors, due to rotation in the two alternating magnetic fields, will tend to reduce the amount of the rotor currents; and the maximum speed will be attained when the E.M.F.'s generated in the conductors, as they cut through the magnetic flux, almost exactly equal and balance the E.M.F.'s due to induction—the difference being such as to allow the very small current to pass which is necessary to overcome bearing friction, iron loss, windage, etc.

The maximum possible speed, if there were no losses in the motor, would be the speed of synchronism; and, since the percentage losses, when running light, are very small, an unloaded induction motor may be said to run practically at synchronous speed.

Thus, if the motor be supposed to have only one pair of poles per phase, as in Fig. 89, and if the frequency of the stator current be 25, then the number of revolutions per minute when running light will be 25 × 60, or 1,500 revolutions, because this is the speed which will make each conductor cut the magnetic flux of any one phase in both directions during the time of one complete cycle of magnetisation. If the speed is N revolutions per minute and the frequency n complete cycles per second, the number of *pairs of poles per phase* will be

$$P = \frac{60 \times n}{N}$$

(78) **Vector Diagram for Induction Motor with Rotor at Rest.**—The condition of things at the moment of switching on the stator current, while the rotor is still at rest, is represented by the diagram Fig. 90.

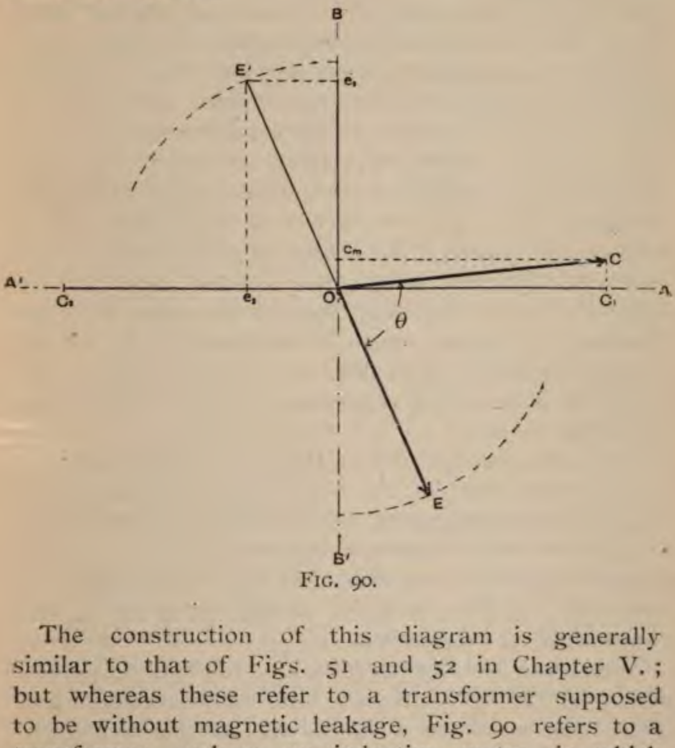

Fig. 90.

The construction of this diagram is generally similar to that of Figs. 51 and 52 in Chapter V.; but whereas these refer to a transformer supposed to be without magnetic leakage, Fig. 90 refers to a transformer—such as an induction motor—in which magnetic leakage plays an important part.

If the rotor is of the squirrel-cage type, the condition is that of a transformer having appreciable magnetic leakage and a short-circuited secondary If, on the other hand, the motor under consideration has a wound rotor with resistances connected in series (for reasons which will be explained later), then the condition is merely that of a transformer with its secondary closed on a definite non-inductive load. The diagram Fig. 90 applies to either case, and, for the purpose of simplifying the construction, we shall assume (as when treating of static transformers) that the ratio of primary to secondary turns is 1 : 1.

It must, moreover, be clearly understood that the diagram only represents the relation of E.M.F. and current in one of the phases, such as that which excites the coils P P (Fig. 89); but exactly the same relation will hold good in the other phase (coils O O, Fig. 89), with the one exception that the diagram, as a whole, must be supposed to be turned through an angle of 90 deg.

The construction of the vector diagram Fig. 90 is as follows:

Draw the vertical line, B′ B, to represent the phase of that component of the total magnetic flux which passes through the rotor. This will induce an E.M.F. $\overline{O\ e_2}$ in the rotor windings, lagging exactly a quarter period behind the magnetism, O B; the magnetising component of the primary current being $\overline{O\ C_m}$, in phase with the magnetism.

Now, since the flux entering the rotor is that component of the total primary flux which produces the *effective* or *resultant* E.M.F., e_2, in the secondary winding (the latter being considered as having no self-induction), the rotor current, $\overline{O\ C_2}$, will be in phase with e_2, and equal in magnitude to $e_2 \div R$, where R is the resistance of the rotor windings.

This secondary current must be balanced by an exactly equal but opposite primary current component, $\overline{O\,C_1}$, the total primary current being $\overline{O\,C}$, obtained by compounding $\overline{O\,C_1}$ and $\overline{O\,C_m}$.*

Of the two components of this total primary current, it is the "wattless" component, $\overline{O\,C_m}$, which produces the flux through the rotor, while the component $\overline{O\,C_1}$ gives rise to the leakage flux which does not pass through the rotor. This leakage flux, being in phase with $\overline{O\,C_1}$, produces an E.M.F. component $\overline{O\,e_3}$ in the primary windings only, exactly one quarter of a period behind $\overline{O\,C_1}$.

By combining this E.M.F. with the induced E.M.F., $\overline{O\,e_2}$—due to that portion of the total flux which passes through the rotor—we obtain $\overline{O\,E'}$, which is the total back E.M.F. in the primary circuit. If we assume the resistance of the primary coils to be negligible, the impressed primary E.M.F. will be $\overline{O\,E}$, exactly equal and opposite to $\overline{O\,E'}$; the power factor, for this particular condition of the rotor at rest, being cos θ, where θ is the angle between the vectors $\overline{O\,C}$ and $\overline{O\,E}$.

(79) **Starting Torque of Polyphase Induction**

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* We are neglecting the losses in the motor, which would make O C slightly greater than as obtained by means of this construction; but, in any case, the "work" component of the total magnetising current is very small in a motor in comparison with the "wattless" component, $\overline{O\,C_m}$. This is due to the necessary air-gap; and, whereas in a well-designed static transformer the true magnetising current might be about 45 deg. in advance of the magnetic flux (see Figs. 51 and 52), in a motor, it would be much more nearly in phase with the magnetism. Thus, when the motor is running light, at synchronous speed, the "work" component of the no-load primary current—required to overcome frictional, windage, hysteresis, and eddy-current losses—would not exceed about 15 per cent. of the "wattless" component, $\overline{O\,C_m}$, which is in phase with the magnetism.

Motor.—Referring still to the diagram Fig. 90, we see that the rotor current C_2 is exactly a quarter period out of phase with the magnetic induction, O B, which enters the rotor. The rotor current cannot, therefore, react upon this field so as to produce a starting torque; and if one phase only of a polyphase motor is connected to the supply, the motor will not be self-starting. But it must be remembered that the other set of coils (O O, Fig. 89) produces a magnetic flux through the rotor, equal in amount, but exactly 90 deg. out of phase with the flux, B B′, due to the coils P P. This flux, being in the direction $\overline{A\ A'}$ (Fig. 90) *is exactly in phase with the current* $\overline{O\ C_2}$ *in the rotor*, as already explained on p. 244.

The starting torque will, therefore, be proportional to *rotor current* × *magnetic flux entering rotor*, or, since $O\ e_2$ is—to a certain scale—a measure of the magnetic flux component which passes through the rotor, we may write

$$\text{starting torque} \propto O\ C_2 \times O\ e_2.$$

In Fig. 90, if the impressed stator E.M.F. is supposed constant, the ends of the vectors $\overline{O\ E'}$ and $\overline{O\ E}$ must necessarily move on the dotted circle described from O as a centre, and it will be an easy matter to study the effect, upon the starting torque, of varying the resistance of the rotor windings, while maintaining constant the supply pressure at the stator terminals.

As regards the running condition, it is evidently an advantage to keep down the resistance of the rotor conductors in order to avoid large $C^2\ R$ losses: but note, at the outset, that *if the rotor conductors were of infinite conductivity, the starting torque would be* *nil.* The smallest amount of magnetic flux passing through the rotor would give rise to an enormous current in the rotor conductors; the result being that the current C_2 would immediately rise to such a value

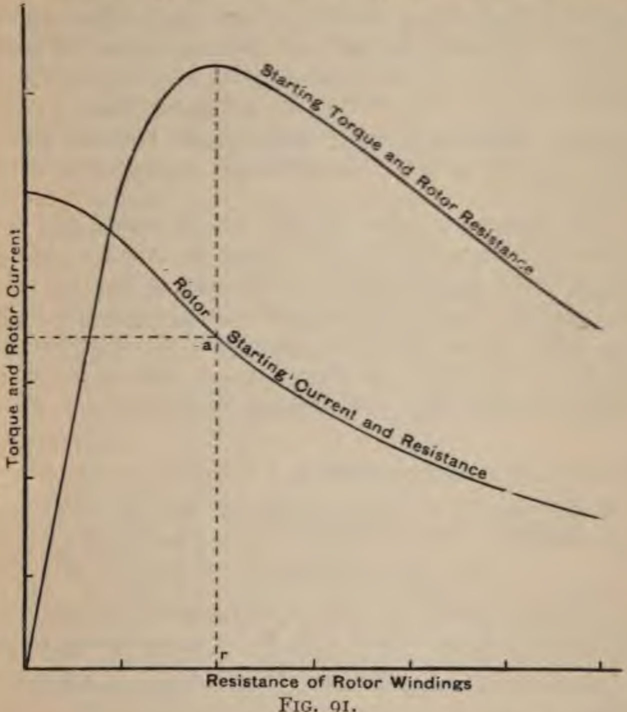

FIG. 91.

as to cause the whole of the primary flux to be leakage magnetism, in the phase $\underline{O\ A}$, thus bringing $\overline{O\ E'}$ to coincide with O B, while $\overline{O\ e_2}$ would be equal to zero.

The curves of Fig. 91 have been plotted from measurements taken off Fig. 90: the upper curve shows how the starting torque reaches a maximum for a definite resistance, r, of the rotor windings, after which, for any further increase of resistance, the torque decreases rapidly. This particular value of the resistance is such as to cause the leakage magnetic flux to be equal to the useful flux which passes through the rotor. In other words, it is the rotor resistance which will make e_3, in Fig. 90, equal to e_2, thus causing the product $e_2 \times e_3$ to be a maximum.

The reason of this is not far to seek, for the induced leakage E.M.F., $\overline{\mathrm{O}\,e_3}$, may be taken as being proportional to the current producing it (seeing that the path of the leakage magnetism is largely through air, which is of constant permeability). The length of the vector $\overline{\mathrm{O}\,e_3}$ is, therefore, a certain multiple of the length O C_2, and, instead of writing

$$\text{torque} \propto \mathrm{O}\,e_2 \times \mathrm{O}\,C_2,$$

we may write $\text{torque} \propto \mathrm{O}\,e_2 \times \mathrm{O}\,e_3$.

With regard to the resistance, this is evidently equal to $\mathrm{O}\,e_2 \div \mathrm{O}\,C_2$, which can also be written

$$\text{rotor resistance} \propto \mathrm{O}\,e_2 \div \mathrm{O}\,e_3.$$

The upper curve in Fig. 91 is obtained by plotting the product $(\mathrm{O}\,e_2 \times \mathrm{O}\,e_3)$ with the corresponding ratio $\frac{\mathrm{O}\,e_2}{\mathrm{O}\,e_3}$; the resultant, O E′ (or O E, the impressed volts), being supposed of constant value.

The lower curve shows the relation between current and resistance; it has been obtained by plotting $\frac{\mathrm{O}\,e_2}{\mathrm{O}\,e_3}$ as abscissæ, with the corresponding values of $\mathrm{O}\,e_3$ as ordinates.

It will be understood that both curves in Fig. 91 will continue to approach zero value as the resistance of the rotor is increased. When the rotor windings are open circuited — *i.e.*, when R = infinity — the torque will evidently be zero, since although the magnetic field through the rotor will have reached its maximum value, there will be no current to react upon it and produce a torque.

(80) **Vector Diagrams for Induction Motor with Armature in Motion.**—When the starting torque, acting upon the armature or rotor, causes the same to revolve, another E.M.F. is generated in the rotor windings. This is the E.M.F. of rotation, due to the cutting of the alternating magnetic field by the rotor conductors : its amount will depend upon the speed of the rotor, and the amount of the magnetic flux cut by the revolving conductors ; it will, moreover, be in phase with the magnetism.

The vector diagram, as drawn in Fig. 90, must, therefore, be modified slightly, and it will now be as drawn in Fig. 92, which applies to the condition of a motor running up to speed.

Let us suppose that the current in rotor — which will be *less* than at starting, on account of the back E.M.F. of rotation — has been reduced to the value represented by the length of the vector $\overline{O\,C_2}$. The corresponding component of the primary current will be $\overline{O\,C_1}$, which produces the leakage field in the direction O B. This gives rise to the leakage induced E.M.F. $\overline{O\,e_3}$, which — as already explained — will be approximately proportional to O C_2, and, therefore, of a smaller value than in the diagram Fig. 90. This enables us to obtain

$\overline{O\ e_2}$ as the induced E.M.F. in rotor windings—the magnitude of the resultant E.M.F. vector $\overline{O\ E'}$ remaining as before.

The magnetising current, $\overline{O\ C_m}$, which has to produce an increased flux through the rotor, will be *greater* than in Fig. 90; it will be approximately proportional to O e_2. The total primary current will be $\overline{O\ C}$, and the impressed E.M.F., $\overline{O\ E}$—drawn equal and opposite to $\overline{O\ E'}$ since we are neglecting the C^2R drop in primary.

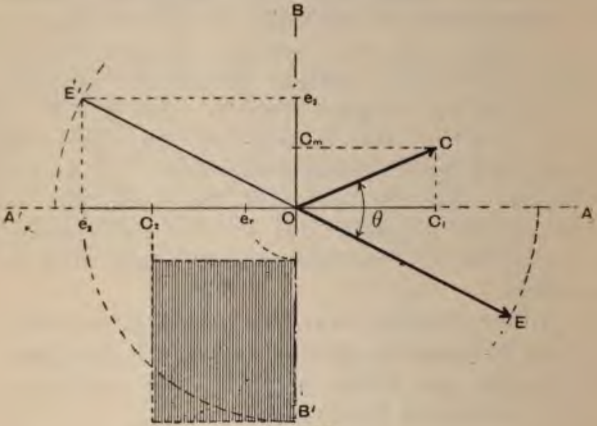

FIG. 92.

Now mark off $\overline{O\ e_r}$, in the same phase as $\overline{O\ C_2}$, to represent the drop of pressure in rotor windings. (The length O e_r will bear the same proportion to O C_2 in Fig. 92 as O e_2 bears to O C_2 in Fig. 90.) This leaves $\overline{e_2\ e_r}$ as the E.M.F. of rotation *due to the conductors cutting through the magnetic lines* *which enter the rotor a quarter period in advance of the flux* O B.

The action, when running, may be summed up as follows:

Phase B of the simple two-phase motor indicated by Fig. 89—when connected to the field coils, P P—produces a magnetic flux, in the direction B′ B, through the rotor. This gives rise to currents in all horizontal closed coils (such as *m*), *which are in phase with the magnetic flux due to the magnetising current of phase* A *in the field coils* O O.

When the armature begins to revolve, a back E.M.F. is produced in the horizontal coils (such as *m*) exactly opposite in phase to the current in these coils. It tends, therefore, to reduce this current, and actually does so, until the difference between the induced E.M.F. e_2 (Fig. 92), due to the magnetism of phase B, and the back E.M.F. due to rotation in the magnetic field of phase A, leaves a resultant or effective E.M.F., e_r, just sufficient to send such a current through the armature as will give the necessary torque.

It will be understood that exactly the same thing occurs a quarter period later, when the same coils occupy vertical positions such as *n*, only in this case the field due to phase A, exciting the coils O O, gives rise to currents in the rotor coils, while the back E.M.F. of rotation in these coils is due to their cutting through the magnetic flux from the poles, P P. The arrangement, in fact, is symmetrical, and each set of stator coils gives rise to currents in the rotor which react upon, and are again affected by, the magnetic flux generated by the other set of coils.

In Fig. 93 the diagram has been drawn for the condition of practically synchronous speed — *i.e.*, motor running light—the only work done being that required to overcome the C^2 R losses in the rotor.*

The whole of the induced E.M.F. will now be in the direction O A′, and $\overline{O\ e_2}$ will be equal in length to the vector of the impressed volts $\overline{O\ E}$, since the leakage flux due to the very small rotor current, $\overline{O\ C_2}$, may be neglected The magnetising current, $\overline{O\ C_m}$, will have reached its maximum value, and the total primary current will be $\overline{O\ C}$, only very slightly in advance of the magnetising component, $\overline{O\ C_m}$ owing to the comparatively small value of $\overline{O\ C_1}$. In a well-designed motor, the total primary current, $\overline{O\ C}$, when running light, would be about one quarter of the full-load current.

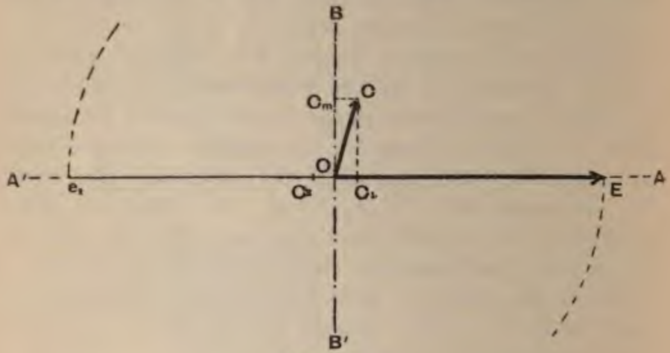

Fig. 93.

Reverting again to Fig. 92, this diagram, if studied attentively, will prove very instructive. It will serve to show that the polyphase motor may be compared with advantage to a direct-current shunt-wound motor, the behaviour of these two types of machines—both when starting up, and when running under load—being in nearly all respects similar.

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* For simplicity of construction, the primary losses are still considered negligible.

The torque exerted by the armature is still proportional to $O\,e_2 \times O\,C_2$; and since $O\,e_3 \propto O\,C_2$, the area of the dotted rectangle $O\,e_2\,E'\,e_3$ is a measure of the torque.

Again, the E.M.F. of rotation, represented by the length $e_2\,e_r$, will depend upon the amount of the flux entering the rotor, and the rate of cutting, or speed of the motor; and since the former quantity is represented to a certain scale by the length $O\,e_2$, we may write

$$e_2\,e_r \propto O\,e_2 \times \text{speed},$$

or,

$$\text{speed} \propto \frac{e_2\,e_r}{O\,e_2}.$$

With regard to the power given out by the motor, this is the product of torque and speed, or

$$\begin{aligned}\text{Power} &\propto \text{torque} \times \text{speed}\\ &= (O\,e_2 \times O\,C_2) \times \frac{e_2\,e_r}{O\,e_2}\\ &= O\,C_2 \times e_2\,e_r\end{aligned}$$

which is a measure of the power, expressed in watts per phase.

This is graphically represented by the shaded rectangle (Fig. 92), and, since the same conditions occur simultaneously in phase A as in phase B, *twice* the watts obtained by the above construction will be the total power developed by the rotor

It is important to know what is the amount of overload that an induction motor will stand—apart from considerations of heating or efficiency — before it will cease to respond to a further increase of load; and this point, together with the reasons for inserting resistances in rotor windings for starting purposes, will be dealt with in the following article.

(81) **Overload Capacity of Induction Motors as Influenced by Magnetic Leakage and Rotor Resistance.**—In the first place it should be noted that, if the ohmic resistance of both stator and rotor windings could be so small as to be considered negligible, the speed of the rotor would be constant at all loads*, and there would be *no limit to the permissible amount of overload* which the motor could momentarily stand (apart from mechanical reasons), *provided there is no magnetic leakage.*

It is the magnetic leakage due to the rotor current—or, more correctly speaking, to the balancing component of the primary current—which determines the maximum value of this current, beyond which any further increase will only lead to a reduced torque; and since the motor will be incapable of responding to a further addition to the external load, it will at this point pull up—*i.e.*, break out of step and come to rest.

If the reader will refer back to Figs. 90 and 91, he will have no difficulty in understanding this.

The curve of Fig. 91 shows how the maximum value of the torque is obtained with a certain definite value of the rotor current, which is something less than the maximum possible value calculated on the basis of a negligible rotor resistance. This amount of current, corresponding to the condition of maximum torque, will depend upon the design of the magnetic circuit; it is that particular value of the rotor current which will make the leakage field equal to the useful field entering the rotor. A larger rotor current would only lead to a weakening of the useful field in which the rotor revolves, and a consequent reduction of the torque, which is the cause of the motor breaking out of step. The importance of a small air-gap, and careful attention to the design of slots and disposition of the windings, will now be readily understood, since, by keeping down the amount of the magnetic leakage, the overload capacity of the motor is increased.*

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* For constant frequency of supply.

Although the upper curve of Fig. 91 refers to the starting torque with rotor at rest, the character of the curve would be similar if drawn to represent the relation between torque and *slip*† when the motor is running under load.

Thus, by connecting a large external resistance in series with the rotor windings, so as to make the total resistance equal to r (Fig. 91), the maximum torque is obtained at starting—*i.e.*, when the rotor is at rest and the slip, therefore, 100 per cent.

This additional resistance must be cut out as the motor runs up to speed, and the actual speed corresponding to the maximum torque will depend upon the amount of resistance left in the rotor windings themselves after the starting resistances have been entirely short-circuited.

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* The clearance between rotor and stator in modern motors may be anything between $\frac{1}{100}$ and $\frac{1}{10}$ of an inch, depending upon the diameter of the rotor. For a rotor 2 ft. in diameter, the clearance could not, for mechanical reasons, be safely made smaller than $\frac{1}{16}$ in In very large motors it might even have to be $\frac{1}{8}$ in. or more.

† The slip being the relative speed of revolving field and rotor, is evidently equal to the difference between the maximum possible speed—*i.e.*, the speed of synchronism, and the actual number of revolutions per minute made by the rotor.

Referring again to Fig. 92, it will be understood that there is a certain value of the rotor current, C_2, for any given motor, which will make the leakage field equal to the useful field, or the leakage E.M.F. component, e_3, equal to the useful component, e_2; each being expressed by the ratio $\frac{E}{\sqrt{2}}$, where E is the total back E.M.F. in stator windings.

The length $O\ e_2$ will be constant for a given value of the vector $\overline{O\ C_2}$; but the portion $O\ e_r$ of this total induced E.M.F. in the rotor, will depend upon the resistance of the rotor windings: it is—as already explained—the E.M.F. required to overcome the resistance, and will, therefore, be equal to $C_2 \times R$, where R stands for the rotor resistance.

Hence, for a given secondary current, such as the critical value corresponding to the condition of maximum torque, the length $O\ e_r$ will be proportional to the rotor resistance. But it is also proportional to the *slip*, because—as explained on p. 257—the rotor speed may be expressed by the ratio $\frac{e_2\ e_r}{O\ e_2}$, which is the same thing as stating that the length $e_2\ e_r$ is a measure of the actual speed, to the same scale as $O\ e_2$ represents the maximum or synchronous speed; thus leaving the *difference* $(O\ e_2 - e_2\ e_r)$ or $O\ e_r$, to stand for the *slip revolutions*, or *relative speed of revolving field and rotor.*

The conclusion to be drawn from the foregoing arguments is that—although the maximum torque exerted by a motor is not affected by the resistance of the rotor winding—the drop in speed which must occur before the maximum torque is reached—*i.e.*, before the machine will cease to respond to a further increase of load—depends upon the rotor resistance, and, indeed, this *drop in speed*, or *percentage slip*, is *directly proportional to the resistance of the rotor windings.*

Example. — If this critical value of the rotor current is 100 amperes per phase, and the induced E.M.F. (e_2) per phase is 12 volts, a rotor resistance of $\frac{12}{100} = {\cdot}12$ ohms would result in the maximum torque being exerted at the moment of starting—*i.e.*, when the speed = 0 or slip = 100 per cent. But if we reduce this resistance to ·06 ohms, the maximum torque will be exerted when the speed is one-half synchronous speed (slip = 50 per cent). If the resistance is only one-sixth of the above value—*i.e.*, ·02 ohms—the breakdown speed would be five-sixths of synchronous speed (slip = one-sixth).

The curves of Fig. 94 have been drawn to illustrate this point.

Curve A would apply to a motor without any external resistance in rotor; it refers to a machine which would break out of step when the slip reaches a value equal to 20 per cent. of synchronous speed, or five times what we have supposed to be the slip at normal full load (4 per cent.): the maximum torque being—in this example—a little over three times the full-load torque.

If the resistance of the rotor windings is increased in the proportion of 5 to 2, the maximum torque will occur at half the synchronous speed (slip = 50 per cent.) (See curve B.)

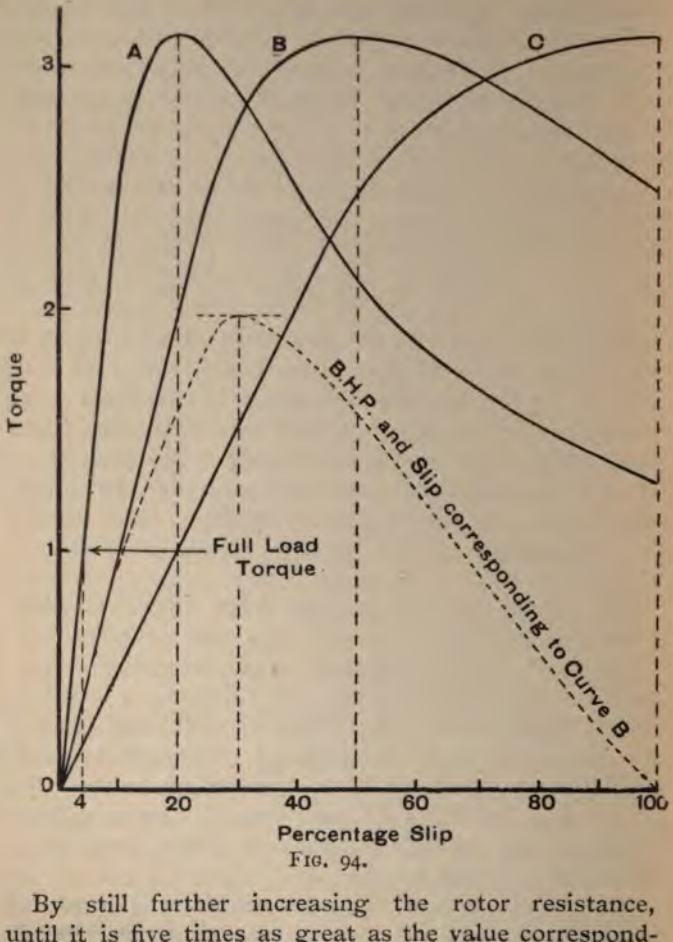

FIG. 94.

By still further increasing the rotor resistance, until it is five times as great as the value corresponding to curve A, we obtain curve C, which shows how the torque would increase with the slip until the maximum value was reached with the rotor at rest (slip = 100 per cent.).

It should be noted that, for small values of the slip, the curves A, B, and C are practically straight lines, and, for this reason, it is always safe to assume that the slip is directly proportional to the torque for all values from zero up to the rated full load of an induction motor.

We have seen that the torque is proportional to the product of current, C_2, and E.M.F. (e_2) induced in rotor, because the latter is evidently a measure of the magnetic field in phase with the rotor current. Hence

$$\textit{torque} \propto C_2 \times e_2.$$

Let us now multiply and divide this expression by e_r (the E.M.F. required to overcome rotor resistance); this makes no difference to the result, and we may, therefore, write

$$\textit{torque} \propto C_2 \times e_r \times \frac{e_2}{e_r}$$

or,

$$\textit{torque} \propto C_2 \times e_r \div \frac{e_r}{e_2}.$$

But note that $C_2 \times e_r$ is the C^2 R loss in the rotor, and $\frac{e_r}{e_2}$ is the slip* which gives us the expression

$$\textit{torque} \propto \frac{C^2 \text{ R loss in rotor}}{\text{slip.}}.$$

The conclusion to be drawn is that, *for a given slip*, the torque exerted by an induction motor is proportional to the *square of the rotor current*. This current will be constant for a given impressed E.M.F. at stator terminals; but if this be varied in amount, the several components, e_3, e_2, and e_r, will be altered in a like manner,* and since the rotor current, C_2, is directly proportional to e_r, it is equally correct to state that, for any given value of the slip, *the torque is proportional to the square of the applied potential difference.*

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* Refer to p. 260 if this is not clear.

Although it is the maximum *torque* which determines the speed at which the motor will break down—*i.e.*, fail to provide the increased turning effort demanded by a further increase of load—it does not follow that this particular speed corresponds with the greatest possible *output* or brake horse-power. The maximum output would occur at a somewhat smaller slip than that which corresponds with the breaking-down point; because, although the *torque* may continue to increase, the *speed* is being reduced, and the maximum value of the product *speed* × *torque* will occur before the torque has reached its maximum possible value.

This is clearly shown by the dotted curve in Fig. 94, which has been plotted to show the relation between speed and power for the particular rotor resistance corresponding to the curve B. It will be seen that the maximum value of this dotted curve occurs with a speed of 75 per cent. (slip = 25 per cent.), whereas the maximum value of the torque curve B occurs at a speed of 50 per cent. of synchronism (slip = 50 per cent.).†

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* For we are still supposing the slip, or the ratio of e_r to e_2, to remain unaltered.

† The overload capacity of a polyphase motor just before falling out of step would, in practice, be something between twice to three times or even four times the rated maximum load.

Another conclusion to be drawn from a study of the foregoing diagrams (see Fig. 92) is that the *speed of the motor is practically independent of the applied potential difference at stator terminals.* If this pressure is increased or reduced, the diagram Fig. 92 will remain as drawn, and it will be merely necessary to suppose the scale by which all the vectors are measured to vary in a corresponding manner. The speed—*i.e.*, the ratio of $e_2\,e_r$ to O e_2—will, therefore, remain as before; and we have here another point in common between the induction motor and the direct-current shunt-wound motor. If the iron in the magnetic circuit be supposed unsaturated, any increase in the pressure at terminals will give rise to a proportionally increased magnetic flux, and, therefore, the *same* speed of rotation of the armature conductors will suffice to produce the necessary back E.M.F. The same argument applies to the induction motor.

(82) **General Conclusions Regarding Magnetic Leakage in Induction Motors.**—The objections to the leakage magnetism, which is produced by the stator coils, but fails to pass through the rotor, are fairly evident.

This dissipation of magnetic flux, which cannot be utilised, leads to a reduction in the overload capacity of the motor, and a diminished torque, for a given slip, at intermediate loads. It also reduces the power factor, and even the efficiency, since it involves increased hysteresis and eddy-current losses in the iron cores and neighbouring metal work.

The primary magnetising current is usually between 20 to 30 per cent. of the normal full-load current, and it should clearly be the aim of the designer to keep it as low as possible.

The chief remedy for leakage troubles in alternating-current motors, and other alternating-current apparatus, is to work with currents of low periodicity;* this permits of a reduction in the necessary number of poles in the motor for a given speed of rotation, and also allows of *higher inductions being used in the iron cores.* This is a distinct gain; and in many cases where improvements in the working of alternating-current apparatus are attributed to the adoption of reduced frequencies, the improvements are not to be accounted for by the reduction of frequency *per see*, but to the fact that the induction in the iron cores has been increased.

The following considerations will make it clear that, with a view to keeping down the percentage leakage loss, it is advantageous to make the magnetic induction in the iron as high as possible; although it must not be overlooked that, for a given frequency, the practical limit in this direction is set by the amount of the hysteresis and eddy-current losses, which increase largely with the higher inductions: but by keeping down the frequency, higher inductions are permissible.

Let B_s stand for the maximum value of that component of the total induction in primary core which passes also through the secondary or rotor windings, and B_l = the maximum value of the leakage component of the total induction in primary core.

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* The expense of working at very low frequencies is, however, great; especially when static transformers are used—as is very often the case—for raising and lowering the pressure at both ends of a transmission line. Induction motors work well at a frequency of 25, but may be used on circuits of periodicities up to 60. The standard adopted by one large Continental firm is 40, as this allows of the use of incandescent lighting, and even arc lamps, off the same circuit.

Then, assuming the resistance of the path of the leakage lines to be constant, we may write

$$B_l \propto \text{ampere-turns tending to produce leakage,}$$
$$\propto T \times C_1,$$

where T = the number of turns in the primary or stator winding, and C_1 = the main component of the primary current. And since the mean back E.M.F. in primary, due to leakage (the vector $\overline{O\,e_3}$ in the last few diagrams), is proportional to $B_l \times T \times n$, where n = the frequency, it follows that we may write

$$e_3 \propto T^2 C_1 n,$$

from which we see that the mean value of the "leakage volts" for any given arrangement of the magnetic circuit is proportional to the square of the number of turns in the winding, and to the current, and to the frequency.

But as it is not the actual value of the leakage E.M.F. with which we are now concerned, it will be advisable to express this as a percentage of the mean E.M.F. available for generating current in the secondary circuit. This last will be proportional to $B_s \times T \times n$; and if we divide the actual "leakage E.M.F." by this amount, we obtain the expression:

Percentage back E.M.F. due to leakage

$$\propto \frac{T^2 C_1 n}{B_s T n}$$
$$\propto \frac{T C_1}{B_s}.$$

It follows from this that, if we consider a transformer on a non-inductive load, the *percentage* back E.M.F. due to leakage is independent of the frequency, but is inversely proportional to the induction in the core enclosed by the secondary winding; and since the induction motor may be regarded as a transformer with short-circuited secondary, the above reasoning is applicable to this special form of alternating-current transformer.

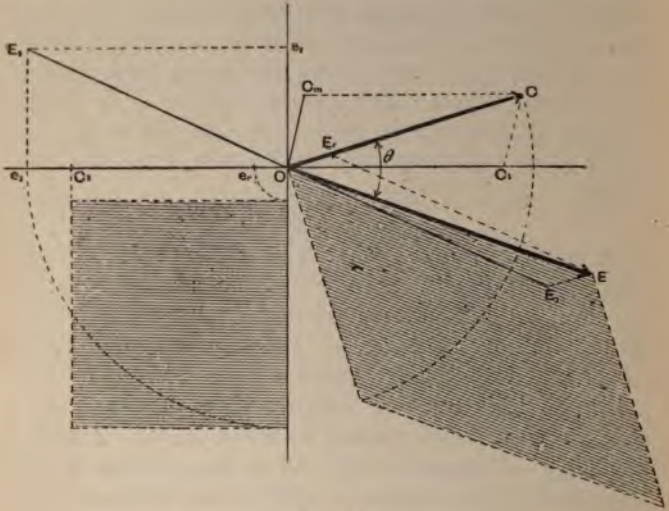

FIG. 95.

(83) **Complete Vector Diagram for Polyphase Induction Motor.**—Up to the present we have supposed the stator or primary winding to be of negligible ohmic resistance, and it will, therefore, be advisable to observe briefly how the problem is influenced by taking this resistance into account.

Fig. 95 is the complete diagram for a polyphase induction motor. It differs in only two respects from the diagram Fig. 92. In the first place, the magnetising current component, $\overline{\mathrm{O\ C}_m}$, has been drawn rather more than 90 deg. in advance of the induced E.M.F., e_2, so as to take into account the small "work" component of the total primary current, required to provide for the hysteresis and eddy-current losses. In the second place, another component, $\mathrm{O\ E}_r$, of the primary impressed E.M.F. has been taken into account. This is the component required to overcome the resistance of the stator windings.

The construction of the diagram Fig. 95 is exactly similar to that of Fig. 92 up to the point of obtaining the primary vector $\overline{\mathrm{O\ E}_1}$ (corresponding to $\overline{\mathrm{O\ E}}$ of Fig. 92); but we have now to add another component, $\overline{\mathrm{O\ E}_r}$,—equal to C × R, where C = the total primary current, and R = the primary resistance—in order to obtain the required potential difference, $\overline{\mathrm{O\ E}}$, at stator terminals.

The introduction of this component of the primary E.M.F. leads to a reduction of the angle θ, and, therefore, an improvement in the power factor; but there is evidently no advantage in increasing the power factor at the cost of additional *wasted* energy. A high primary resistance cannot be otherwise than objectionable; it absorbs a certain amount of energy which would otherwise have been available for doing useful work.

(84) **Efficiency of Polyphase Motors.**—The two shaded parallelograms in Fig. 95 respectively represent the power which is supplied to the primary terminals, and that which is transmitted to the shaft by the revolving rotor.

The power, per phase, supplied to the primary terminals is E × C × cos θ, and it is proportional to the area of the parallelogram constructed on one of the vectors, $\overline{\text{O E}}$, with the other vector, $\overline{\text{O C}}$, moved round through an angle of 90 deg.

The power given out by the rotor is $C_2 \times e_2$ *less* the C^2 R losses in the rotor windings, or $C_2 \times (e_2 - e_r)$, and the *efficiency* will be the ratio of this last quantity to the power put into the motor at primary terminals.

The full-load efficiency of commercial polyphase motors will vary between 75 per cent. for small sizes and 90 per cent. for large sizes; and it will fall off somewhat on an appreciable overload.

The following table, compiled by the author from figures referring to a large number of three-phase motors of various makes, may be of use as indicating the average full-load efficiency and power factor likely to be obtained with this class of machine:

B.H.P.	Efficiency.	Power factor.
1	·7	·7
2	·76	·76
4	·81	·8
7	·85	·81
10	·87	·82
20	·89	·84
50	·91	·87
100	·93	·9

The figures given for the power factor may be considered as being somewhat on the low side, as they refer to machines working at a periodicity of about 50; and, with lower frequencies, such as

25 cycles per second, slightly better results might be expected.

(85) **Methods of Measuring the Slip of Induction Motors.**—One of the most important factors to be measured when testing an induction motor is the slip. A large slip for a given load is an indication of large losses in the rotor windings. We have already seen how the slip, for a given torque, is proportional to the resistance of the rotor windings. It has also been explained in connection with the vector diagrams (see, for instance, Fig. 95) that the slip, expressed as a fraction of the synchronous speed, is $\frac{e_r}{e_2}$, where e_r stands for the volts required to overcome rotor resistance, and e_2 is the E.M.F. induced in the rotor (the difference, $e_2 - e_r$, being the back E.M.F. due to rotation in the magnetic field).

Now, it is evidently equally correct to write

$$\textit{percentage slip} = 100 \times \frac{C_2\ e_r}{C_2\ e_2},$$

which amounts to defining the slip as the ratio of rotor copper losses to the total power supplied to rotor by induction from the primary circuit.

If, therefore, we know the rotor resistance and current, we can calculate the slip. If the rotor is of the squirrel-cage type, the equivalent resistance can be ascertained as follows:

Clamp the rotor so as to prevent rotation: supply a low pressure to the stator terminals until the required primary current is obtained: measure the total power supplied to the primary, and deduct the calculated C^2 R losses in stator windings. This leaves a quantity representing with sufficient accuracy the C^2 R losses in the rotor, corresponding to a definite primary current.

Evidently the slip could be measured by taking careful readings of the speed when running light (synchronous speed), and, again, when running under load. The difference would be the slip: but as this is a very small quantity relatively to the two speed measurements, the inaccuracy and disadvantages of such a method are obvious.

Sometimes a small synchronous motor is used to give the exact speed of synchronism at the moment when a reading is taken; and it would be an easy matter to devise a counter which would directly indicate the *difference* between the speeds of the synchronous motor and the motor under test; indeed, it is not improbable that some such device has actually been put upon the market.

But a number of satisfactory methods of measuring the slip will suggest themselves to anyone with a little ingenuity who cares to study the question. It will be sufficient if we confine ourselves to briefly describing two very useful methods.

Imagine a small contact stud fixed to any convenient point on the rotor, in such a position as to momentarily close an electric circuit once during each revolution. This circuit should be connected, through a voltmeter, to the same source of supply as the stator windings (see Fig. 96). If the motor were running at synchronous speed, the indication of the voltmeter would be steadily of the same value, since the local circuit would always be closed at exactly the same instant in the cycle of the supply pressure. But if the machine is running at something less than synchronous speed, the pointer of the voltmeter will take up a swinging motion. This is due to the local circuit being closed each consecutive time slightly later in the E.M.F. cycle; and every double oscillation of the pointer will

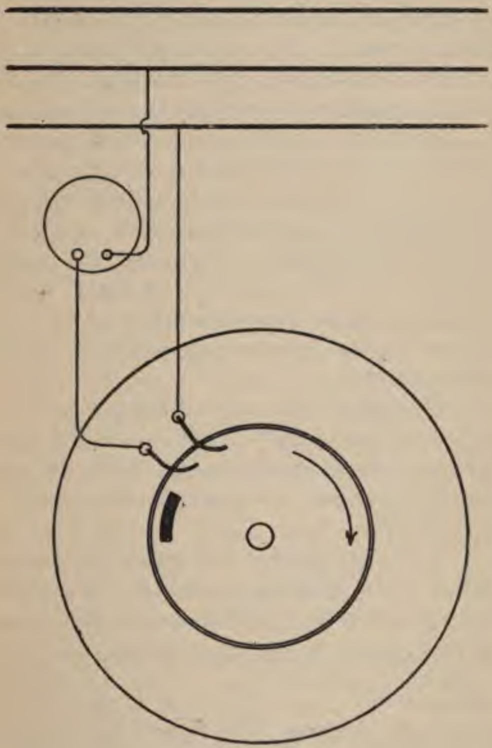

FIG. 96.

correspond to one complete wave of alternating E.M.F., thus indicating that — during the time of one such double swing — the rotor is behind the position it would have occupied if synchronous, by twice the angle subtended by two consecutive poles of the same phase. By counting the number of double swings per second, and dividing this by the (known) frequency of supply, we get the percentage slip.

Methods based on this principle have given most satisfactory results. Sometimes a telephone receiver is used in place of the voltmeter, with advantage.

Another excellent and simple method of measuring the slip consists in fixing to the shaft of the motor a circular disc having equally spaced black and white segments painted upon it, the number of each colour corresponding to the number of pairs of poles per phase in the stator.

If this is illuminated by an arc lamp connected to the same source of supply as the stator, the disc will appear to revolve backwards; and the number of apparent revolutions in a given time will be a measure of the slip.

Thus, if p = the number of pairs of poles per phase, and n = the frequency, and if the number of apparent revolutions be counted during a time equal to $100 \times \frac{p}{n}$ seconds, this number would represent the percentage slip.

As an example, suppose $p = 8$ and $n = 40$. Now count the number of apparent revolutions of the disc during a time equal to

$$\begin{aligned} 1{,}000 \times \frac{8}{40} &= 200 \text{ seconds}, \\ &= 3 \text{ minutes } 20 \text{ seconds}; \end{aligned}$$

let us say that 38 apparent revolutions are observed in this time; then the slip corresponding to the particular load conditions under which the observation was made, would be 3·8 per cent.*

(86) **Circle Diagrams.** — Although the vector diagrams previously used are useful for explaining the principles underlying the behaviour of induction motors, the construction for obtaining the primary current and power factor for various values of the rotor current is lengthy, and, moreover, in the case of Fig. 95, where the primary resistance is taken into account, a correction has to be made so as to reduce all results to a constant value of the impressed volts, E.

It is true that this correction merely amounts to a simple proportion sum, determining the *scale* by which the various vectors should be measured; but, as will be seen, the circle diagrams (originally introduced by Mr. Alex. Heyland) have much to recommend them, and are very convenient to use.

In its simplest form, the circle diagram is as drawn in Fig. 98. This applies to an induction motor in which all C^2 R losses in the copper, and hysteresis and eddy-current losses in the iron, are supposed to be so small as to be negligible.

The corresponding vector diagram, in the form with which we are already familiar, has been drawn on page 276 (see Fig. 97).

It will be seen that Fig. 97 is practically identical with Fig. 92 (p. 254), but it has been re-drawn so as to facilitate direct comparison with the circle diagram.

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* For a description of an ingenious direct-reading indicator on the above principle, see article by Mr. C. V. Drysdale in the *Electrician* of Aug. 25, 1905.

There is a definite relation between the length of the rotor current vector, $\overline{\mathrm{O\,C}_2}$, and the magnetising current component, $\overline{\mathrm{O\,C}_m}$, which may be deduced as follows:

The secondary E.M.F., or volts induced in the rotor, will be proportional to the amount of the magnetising current,* or

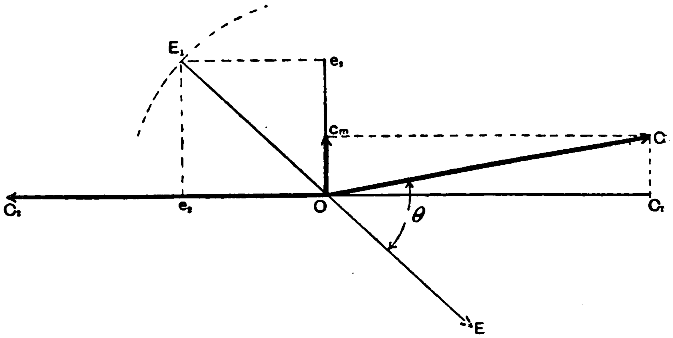

FIG 97.

$$e_2 = \mathrm{C}_m \times k_m,$$

where k_m is a constant. Moreover, the leakage volts, e_3, are proportional to the rotor current, C_2, or

$$e_3 = \mathrm{C}_2 \times k_2,$$

where k_2 is another constant.

---

* The magnetic circuit is supposed to be of constant permeability, and, therefore, the amount of the magnetic induction is proportional to the current producing it. This assumption is justified on account of the comparatively low inductions used in the iron, and the relative importance of the air-gap in the total resistance of the magnetic circuit.

A third condition is that

$$e_2^2 + e_3^2 = E^2,$$

where E stands for the impressed voltage, and is, therefore, a constant. We shall assume, for convenience, that E is equal to unity.

This enables us to write

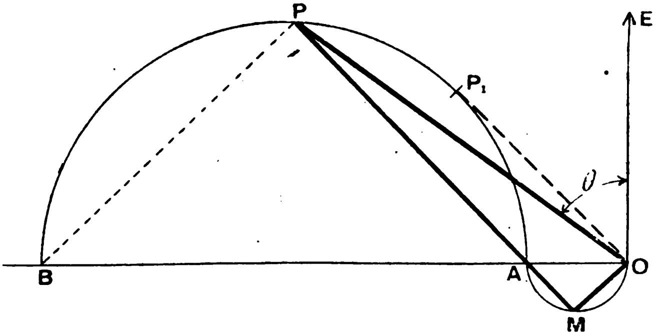

FIG. 98.

$$e_2^2 + e_3^2 = 1$$

or,
$$C_m^2 \; k_m^2 + C_2^2 \; k_2^2 = 1.$$

Hence
$$C_m^2 = \frac{1 - C_2^2 \, k_2^2}{k_m}.$$

It will simplify matters if we put figures in place of the two constants.

Let us suppose that $k_m = \frac{1}{2}$ and $k_2 = \frac{1}{10}$, then

$$C_m^2 = 4 - \frac{4}{100} \, C_2^2.$$

There is, of course, another equation which gives us the total primary current, C, and this is

$$C^2 = C_m^2 + C_2^2$$

because the triangle O C C_m (Fig. 97) is a right-angled one, of which O C is the hypothenuse.

Now consider the diagram Fig. 98. Draw the vertical line O E to represent the phase of the applied E.M.F. and on O B—at right angles to O E—make O A equal to the maximum possible value of the magnetising current (O A = 2 E, since $e_3 = 0$ and $e_2 = E$) and O B equal to the maximum possible value of the rotor current (O B = 10 E, since $e_2 = 0$ and $e_3 = E$).

On A B and O A as diameters, describe the semi-circles as shown. Then, for all intermediate values of the primary current, the end, P, of the primary current vector, $\overline{OP}$, will lie on the larger circle. The line $\overline{PM}$ represents the rotor current, while $\overline{OM}$ is a measure of the magnetising component of the total primary current, $\overline{OP}$. The angle E O P is the angle of lag, the cosine of which is the power factor; and the phase relations of the other two current components are also correctly shown relatively to the pressure vector, $\overline{OE}$.

This diagram correctly reproduces the conditions of Fig. 97, although in a somewhat different form. That the same relations still hold good between the various current vectors can be proved as follows:

The square of the rotor current is C_2^2 or $(P\,M)^2$, and it may be written

$$(P\,M)^2 = (P\,A + A\,M)^2.$$

But, owing to the particular values of the constants k_m and k_2 that we have assumed,

$$A\,B = 4\,O\,A,$$

and $$P\,A = 4\,A\,M$$

(since the triangles O M A and B P A are similar). Hence

$$C_2^2 = (\mathrm{P\ M})^2 = (5\ \mathrm{A\ M})^2,$$

and, to express this in terms of the magnetising current, C_m (or O M), we have

$$(\mathrm{O\ M})^2 + (\mathrm{A\ M})^2 = (\mathrm{O\ A})^2$$

or,

$$C_m^2 = (\mathrm{O\ A})^2 - (\mathrm{A\ M})^2$$

$$= (\mathrm{O\ A})^2 - \left(\frac{\mathrm{P\ M}}{5}\right)^2$$

Now, with our assumption of $E = 1$ and $C_m = 2\,e_2$, the maximum value of the magnetising current (when $e_2 = E$) will be 2. Hence

$$C_m^2 = 4 - \frac{C_2^2}{25},$$

which is exactly the same relation as deduced from the vector diagram Fig. 97.

It is interesting to note that the maximum possible power factor—*i.e.*, the smallest angle θ—occurs when the primary current vector is tangential to the large semicircle, as indicated by the dotted line, O P_1, and this shows very clearly the bad effect, as regards the power factor, of a large air-gap and resulting magnetising current, O A, and of bad design generally, leading to a large leakage field, or increased ratio of O A to O B.

If a larger air-gap were to reduce the leakage magnetism due to the rotor current proportionately to the increase in the magnetising current, the maximum possible power factor would not alter; but this is not what one would expect, and, indeed, from actual tests made by Mr. B. A. Behrend, the effect of enlarging the air-gap of an experimental induction motor was to increase the magnetising current in the ratio of 3 to 7, while the short-circuited rotor current (rotor at rest) only increased in the ratio of 3 to 3·4; thus showing the importance of keeping the air-gap as small as mechanical considerations will admit.

(87) **Circle Diagram Taking Losses into Account.**—If we are to take into account the iron losses and the resistance of the stator coils, the diagram Fig. 98 would not be strictly correct; but it will be found that the end, P, of the current vector, $\overline{O\ P}$ still moves upon a circle, as the load on the motor is varied.

Referring to the complete vector diagram Fig. 95 (p. 268), it will be noted that the impressed E.M.F., E, is no longer exactly equal and opposite to the total back E.M.F., E_2; but, owing to the resistance of the primary winding, it is greater than E_2, and the phase angle, θ, between total current, C, and impressed volts, E, is *smaller* than it would be if the primary losses were inappreciable.

The other point of difference between Figs. 95 and 97 (or Fig. 92) is that the magnetising component ($\overline{O\ C_m}$) of the primary current, is now more than 90 deg. in advance of the rotor current, $\overline{O\ C_2}$; or, in other words, the angle C C_m O in the triangle O C C_m is something greater than a right angle.

We may, however, assume that this angle—*i.e.*, the phase difference between magnetising and rotor currents—remains constant under all conditions of load, and the two chief differences between the circle diagram Fig. 98 and the corrected diagram Fig. 99 may be summed up as follows:

1. The vector of the no-load magnetising current ($\overline{\text{O A}}$), in the corrected diagram, is no longer at right angles to the pressure vector ($\overline{\text{O E}}$), but makes an angle E O A with the vector $\overline{\text{O E}}$, which is less than 90 deg., and corresponds with the actual measured power factor when the motor is running light.

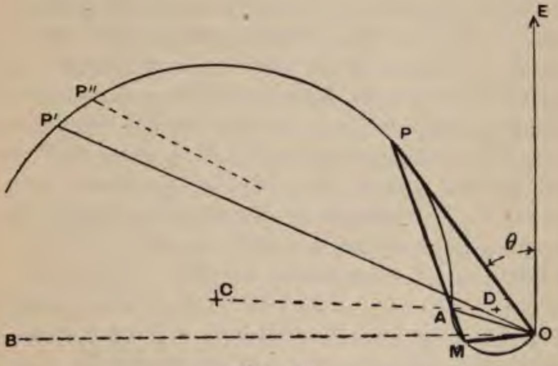

FIG. 99.

2 The constant angle of phase difference, (O M P), between rotor current and magnetising component of primary current, is somewhat greater than 90 deg., and will depend upon the iron losses in the motor.

The diagram Fig. 99 has been constructed from measurements taken off Fig. 95, this latter diagram having been re-drawn for various values of the rotor current, and the results all brought to a common basis of comparison by supposing the impressed volts $\overline{OE}$ to remain of a constant value. It will be seen that the centres, C and D, of the two semicircles no longer lie on the line O B, at right angles to O E, but above this line;* otherwise the diagram is very similar to Fig. 98.

The maximum possible value of the primary current will be O P′, which corresponds to the condition of starting (rotor at rest), and the exact position of the point P′ on the circle will depend upon the amount of resistance in the *rotor* windings. The vector $\overline{OP'}$ has been drawn on the assumption of a reasonably small rotor resistance—which would result in a loss of pressure somewhat less than that represented by the length O e_r in Fig. 95—but, with an increased rotor resistance, the final position of the primary current vector would be on the circle between P′ and P; as, for instance, at P″.

To construct such a diagram which shall represent the behaviour of an actual motor, it is only necessary to have the same particulars as would be required for the construction of a vector diagram of the kind drawn in Fig. 95 (because the circle diagram is merely another form of this general diagram); these particulars are:

1. The no-load current and pressure and true power.

2. The current and pressure and true power with rotor short-circuited, and at rest.

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* In Mr. Heyland's original description of his circle diagram (see *Eclairage Électrique*, July 14, 1900) he makes the centre of the large circle fall on the line O B, at right angles to the pressure vector; but this will not be found to give results in strict accordance with the data obtained from actual tests.

3. The resistance of the primary windings per phase.

It is not proposed to take up any further space here in order to explain how the diagram may be drawn from the above particulars; but if the reader cares to follow the matter further, he is referred to Note 4 at the end of the book.

(88) **Methods of Starting Induction Motors.**—To start an induction motor by merely closing the switches on the supply circuit, and so instantaneously connecting the terminals to the full supply pressure, is evidently unsatisfactory, on account of the very large rush of current which would occur; and, if the motor has to run up under load, this abnormally large current might continue for an appreciable time.

Such a method of starting would only be allowable in the case of very small motors, although it is sometimes adopted for machines as large as 10 b.h.p.

By reducing the pressure across terminals at the moment of switching on, by the introduction of resistances, choking coils, or transformers in the primary circuit, any size of motor, with short-circuited rotor, can conveniently be started up, *provided no great starting torque is required;* that is to say, provided the motor has not to start up against a heavy load.

The method almost universally adopted for starting motors under load consists in providing the motor with a *wound* rotor, the three sections of the winding having their starting ends connected at a common junction, while their finishing ends are taken to three slip rings, from which the current is collected by brushes in the usual manner. These brushes are connected to the three sections of a variable non-inductive resistance, which may be of the liquid type, or made up of wire spirals joined up to a simple type of controller having (say) half a dozen contacts on each of the three regulating arms. The other ends of the three regulating resistances are all permanently connected together.

When starting up the motor, the controller handle is moved to the position where the three rotor circuits are *open*—*i.e.*, the resistance in series with the windings is infinity. The stator current is then switched on under full pressure, and the controller handle moved forward so as to close the rotor circuit through the resistances, which are gradually cut out as the machine runs up to speed, until, when full speed is attained, all external resistance is cut out, and the rotor windings are left short-circuited upon themselves. These operations are equivalent to inserting a starting resistance in series with the armature of a shunt-wound direct-current motor; and almost any starting torque can be obtained in this manner, as will be evident if the reader has carefully followed article 79 and understood the meaning of the curves in Fig. 91.

It has been shown how, by suitably increasing the rotor resistance, the torque, *with rotor at rest*, may be increased up to the maximum torque which the motor can exert under any conditions of working; whereas, if the resistance of the short-circuited rotor is small (which it should be, for the most economical *running* conditions), the torque at starting, without the introduction of some external resistance, will not be equal to the maximum possible torque, although the current taken from the

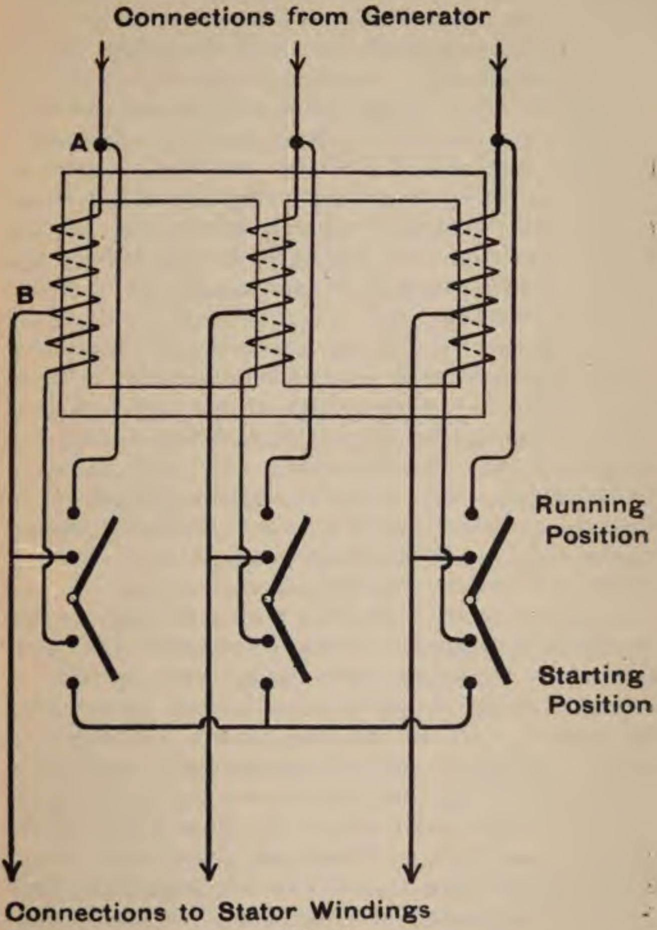
Connections from Generator
A
B
Running Position
Starting Position
Connections to Stator Windings

FIG. 100.

supply mains will be *greater* than that which passes when a starting resistance is inserted.

It has also been explained how this effect is due to phase displacement between the rotor currents and the magnetic field upon which they react; and it is, therefore, not proposed to dwell any longer upon this, the most general and satisfactory method of starting.

With regard to the other method already referred to—namely, reducing the supply pressure at starting before connecting the motor to the circuit—perhaps the most approved way of effecting this is by means of auto-transformers.

Fig. 100 shows the connections to a three-phase auto-transformer, such as would be used for a three-phase induction motor up to about 15 b.h.p. size; the arrangement for larger motors being similar, but it is then advisable to arrange for two or more tappings from each of the transformer windings, so that the pressure may be increased gradually, instead of throwing over suddenly from the reduced starting pressure to the full line pressure.

As shown in Fig. 100, the auto-transformer consists of the usual magnetic circuit of laminated iron; but, instead of there being two distinct sets of coils as in the ordinary transformer, each limb carries only a single continuous winding, with a connection tapped off at a suitable intermediate point. By carefully following the connections on the diagram, it will be seen that, when the three poles of the starting switch are closed on the lower set of contacts, the three transformer windings have their free ends connected to a common "neutral" bar; and they will consequently act merely as choking coils, or as the primaries of a three-phase transformer with open secondary circuit. It will be noticed, however, that the tappings from somewhere about the centre of the windings go directly to the terminals of the motor, which, instead of receiving the full line pressure, only get a reduced pressure, depending upon the ratio of the number of turns comprised between the points A and B, and the total number of turns on each limb of the magnetic circuit. When the rotor has attained a fairly high speed, the switch is thrown over to the upper, or running position, thus short-circuiting the portion A B of the winding, and leaving the motor terminals connected directly to the supply.

It has already been shown how, if a large starting torque is required, it is necessary to increase the rotor resistance beyond the value which would prove economical, or even possible, for continuous running. Many ingenious methods have been suggested for accomplishing this end without the use of sliding contacts and external resistance coils; but nearly all the methods suggested are costly, even if they do not lead to undesirable complications. It is not a difficult matter to devise a machine having two rotors coupled together, which would be capable of exerting a large starting torque. One suggested arrangement is to have two rotors, one of high resistance and the other of low resistance, coupled rigidly together, and provided with a movable stator. Such a machine would be started up with the stator poles over the high-resistance rotor, and as the speed increased the stator would be made to slide in a direction parallel to the shaft until, when full speed is attained, it would occupy a position immediately over the portion of the rotor with the heavy, or low-resistance windings. A machine of this type, however, is only likely to be of commercial value in very exceptional cases.*

The induction motor with "squirrel-cage" rotor is undoubtedly the simplest and the least likely to give trouble when entrusted to unskilled hands, and the method of starting up such a machine by merely closing the switch connecting the stator windings to the supply has much to recommend it. It is possible to start comparatively large motors in this way, if the abnormal rush of current when switching on is not too serious an objection: moreover, for certain conditions, where the highest efficiency is not important, and where heating troubles are not likely to arise, it is possible to deliberately design the rotor with a comparatively high resistance—this being usually provided in the end rings to which the copper conductors are connected.

It would also be helpful, in motors of this description, to provide a flexible or spring coupling between the rotor shaft and the load, as this would permit of the rotor getting up a certain velocity which would increase the effective turning moment at the instant of starting.

In conclusion, it will be seen, on referring to article 82 (p. 265), that, by keeping up the induction in the iron, the starting torque, for a given arrangement of the windings, will be greater than with low inductions. If the best starting results are to be obtained, particular attention must be paid to the question of magnetic leakage (which must be kept as small as possible), and the resistance of the *stator windings* must be small, otherwise the effective E.M.F. which determines the amount of the magnetic flux may be considerably less than the potential difference at stator terminals.

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* For a brief summary of some suggested methods, together with a description of a machine devised by Mr. Frank Lewis, see the *Electrical Review*, p. 1,044, June 30, 1905.

(89) **Speed Regulation.**—The analogy between the induction motor and the shunt-wound direct-current motor, has already been pointed out; both are essentially constant-speed machines, and it is usually uneconomical to use them for variable-speed work. A reduction of the supply voltage does not meet the case, because, although the required back E.M.F. is smaller than with the normal supply voltage across terminals, the magnetic field through the armature is also reduced, and approximately the same speed is required to produce the (lower) back E.M.F.

There are two ways of regulating the speed of a shunt-wound direct-current motor: (1) inserting resistance in series with the armature winding, while the full voltage is maintained at the terminals of the field windings; (2) inserting resistance in series with the field windings, while the normal supply voltage is maintained across the brushes.

By method (1) the magnetic induction remains constant, but the necessary back E.M.F. will depend upon the value of the resistance in series with the armature. If this is such as to absorb, say, 25 per cent. of the total supply volts, the required back E.M.F. will now be 75 per cent. of what it would be under the full voltage; the result being that the machine will run at three-quarters of the normal speed. This method is evidently wasteful, since the resistance must necessarily dissipate the whole of the power which is not given out by the motor through the shaft. For a given turning effort—which depends upon the product *armature current* × *magnetic field*—the power taken from the mains at constant voltage is independent of the motor speed, whereas the brake horse-power is directly proportional to the speed; and the wasted horse-power must, therefore, be in direct proportion to the speed reduction.

By method (2) the magnetic induction through the armature is varied, but the required back E.M.F. remains constant. The speed will therefore vary inversely as the strength of the magnetic field. This method is evidently less wasteful than the first; but, for considerable speed variations, difficulties arise owing to the main field being necessarily weak for the higher speeds, while the leakage field due to the armature currents is relatively large.

Let us now look at the induction motor. Method (1) is almost exactly reproduced by inserting resistance in the *rotor* windings; while the variation in the induction, by method (2) would be effected by altering the *frequency* of the supply. (It is the frequency that—for a given impressed E.M.F.—determines the value of the induction; and the *speed* of the rotor, when running light, will therefore vary directly as the frequency.)

In article 77 (p. 246) it was shown that, if

N = the speed in revolutions per minute;

n = the frequency;

p = the number of pairs of poles per phase in stator,

$$p = \frac{60 \times n}{N}$$

from which we see that the speed will vary directly as the frequency, and inversely as the number of poles. If a simple form of frequency transformer could be devised, by means of which the frequency of the supply circuit could be gradually altered, this would make it possible to conveniently regulate the speed of induction motors, within certain limits, depending upon the amount of the leakage field with the higher frequencies (or low inductions), and the permissible value of the induction in the iron with the lower frequencies and speeds.

By changing the number of stator poles, it is possible to design a motor to run at two or more different speeds; and tests made on such motors have shown the method to be practicable; but it involves some complication in the stator windings, and the use of special switches with connections so arranged that, by combining the coils in various ways, the number of poles may be altered. Moreover, since an induction motor is usually designed, in the first instance, with as large a number of poles as the diameter of the rotor will allow of, any increase in this number, for the purpose of giving a reduced speed, will lead to a corresponding increase in the magnetic leakage, with its attendant disadvantages.

Returning to method (1) as applied to induction motors, the effect of introducing resistance in the rotor windings has already been fully discussed (see article 79, p. 250). It must, however, be clearly realised that the reduction in speed is only obtained by wasting the balance of the power in the resistances, exactly as in the case of the direct-current motor with regulating rheostat in series with the armature.

Example.—Consider an induction motor of 100 brake horse-power, the efficiency of which is ·9 at full load; and assume that it is required to reduce the speed 20 per cent. The power supplied to the motor at full load is

$$100 \times 746 \times \frac{10}{9} = 83 \text{ kw. (approx.)}$$

and the power absorbed by the resistances inserted in rotor winding, for a speed reduction of 20 per cent. must be

$$\frac{83 \times 20}{100} = 16{\cdot}6 \text{ kw.}$$

In order to calculate the ohms required per phase, it is necessary to know the value of the full-load current — *i.e.*, the current corresponding to the maximum torque to be exerted by the rotor. Let us assume a star-wound rotor with 300 volts between any two of the three slip rings on open circuit. Then if 80 kw. be taken as the power put into the rotor, the current in each of the three arms of the winding will be

$$C = \frac{80{,}000}{\sqrt{3} \times 300} = 154 \text{ amperes.}$$

Now, since the total power dissipated in the resistance is 16·6 kw., the watts absorbed by each of the three sections will be

$$\frac{16{,}600}{3} = 5{,}530.$$

Hence $C^2 R \text{ (per phase)} = 5{,}530,$

which gives us for the resistance of each section of the resistance

$$R = \frac{5{,}530}{(154)^2} = \cdot 233 \text{ ohms.}$$

If, instead of absorbing the difference of pressure in a resistance, it were possible to insert a back E.M.F. in the rotor windings, from an outside source, exactly the same result would be obtained. Such a method has, indeed, been suggested, but since some form of commutator is required, involving the use of a rotor wound somewhat in the same manner as a direct-current motor armature, the machine becomes more costly, and loses its peculiar advantage of mechanical simplicity owing to the absence of a commutator.

(90) **Reversing Direction of Revolution.**—The direction in which a polyphase induction motor will run is determined by the direction of rotation of the magnetic field. By reversing the direction in which the field revolves, the direction of running of the rotor will also be reversed. In fact, a simple form of throw-over switch in the connections to the stator terminals is generally all that is necessary.

If the supply is three-phase, it matters not whether the coils are star or mesh connected, but, provided the stator is fed by only three wires, the effect of crossing the connections to any two of the terminals will cause the reversal of the rotating field, and, therefore, of the rotor.

(91) **Recapitulation.**—We have now studied the induction motor by means of vector diagrams, and seen how this type of machine may be considered as a special case of the alternate-current transformer; the points of difference being:

1. That the secondary winding is free to revolve in the field produced by the primary, and so generate a second E.M.F. called the E.M.F. of rotation which—by acting against the induced E.M.F.—limits the flow of current until this is just sufficient to produce the required torque.

2. Owing to the necessary mechanical clearance, or air-gap between primary and secondary cores, the magnetising current is greater than in the ordinary closed-circuit transformer; moreover, the total primary current, when running light, is more nearly in phase with the true (or "wattless") magnetising component, because the "work" component due to the hysteresis and eddy-current losses is comparatively small.

3. The magnetic leakage is much greater than in a well-designed static transformer. It is the aim of the designer to keep this down as far as possible, because a large leakage flux means small overload capacity, although when the machine is running lightly loaded—*i.e.*, *when the rotor currents are small*—nearly all the magnetic flux passes through the rotor, and the leakage is in any case small.

This leads up to the consideration of what is the best shape of slot through which the stator windings are threaded. In this country, and on the Continent, it is usual to completely close the slots, leaving an exceedingly thin metal bridge adjoining the air-gap; this being effected by punching circular holes or elongated slots with semi-circular ends, very near to the periphery of the iron stampings. In America it is not unusual to provide a small gap at the top of the slot, the object being to reduce the magnetic leakage This has little or no effect upon the running qualities of the motor (except as regards the overload capacity); but it admits of a *greater starting torque* being obtained, or the same starting torque with a reduced primary current, owing to the fact that a larger proportion of the total primary flux passes through the rotor. It must, however, not be overlooked that open slots — although they may reduce the leakage—generally increase the resistance of the magnetic circuit through the rotor, and consequently the amount of the "wattless" magnetising current. The effect of a large air-gap between stator and rotor cores, is also to lower the power factor—since it involves a large "wattless" component of the primary current—but it does not appreciably alter the amount of the starting current—*i.e.*, the total primary current with rotor short-circuited and at rest. This will depend mainly upon the disposition of the primary coils and the shape of the slots in which they are wound. It is the leakage between the poles that is the principal factor in determining the amount of the starting current; and even if the rotor were entirely removed from the machine, it does not follow that the current taken by the primary coils would appreciably alter in amount. The stator would act merely as a choking coil and take a current depending upon the magnetic resistance of the leakage paths; this current being small in a badly designed machine, and comparatively large in a machine designed for a heavy overload and large starting torque.

In conclusion, although the writer prefers to consider the polyphase induction motor as a modified transformer with a closed secondary free to revolve in two alternating magnetic fields at right angles to each other, and with a phase difference of 90 deg., it does not follow that it is incorrect or more difficult to treat the subject from the "rotating field" point of view. Indeed, if the reader will again turn to article 28, Chapter III., he will probably find that his conception of a short-circuited rotor being dragged round by the revolving field is clearer to him than before reading the present chapter.

CHAPTER VIII.

ASYNCHRONOUS GENERATORS, COMPENSATED INDUCTION MOTORS, AND ROTARY CONVERTERS.

(92) **The Polyphase Induction Motor used as a Generator.** — It has been explained in the last chapter how, when the load is put on an induction motor, the "slip," or the *difference in speed between rotor and magnetic field,* increases; the reason being that, with an increased torque, a greater rotor current is required to meet it, and—with an approximately constant speed and strength of the rotating field—this increased current can only be obtained by a lowering of the back E.M.F. in the rotor conductors such as will be the immediate result of an increased "slip."

When the motor is running light, the slip is small—that is to say, the rotor is running nearly, but not quite, at synchronous speed — this being due to the fact that a small rotor current must necessarily flow, or sufficient torque would not be produced to overcome bearing friction, windage, etc.

Imagine a small auxiliary motor coupled to the shaft, and supplied with power from an outside source. This motor could be arranged to increase the speed of the rotor until this was exactly equal to the speed of synchronism: the rotor current would then be zero, but the auxiliary motor would be supplying the power to overcome the light-load losses referred to above.

Suppose, now, that the speed of the auxiliary motor be increased so as to drive the rotor of the polyphase machine at a speed *above synchronism—i.e., greater than that of the rotating field.* The result will be that the E.M.F. generated in the rotor conductors, due to the cutting of the magnetic field, will now be *greater* than the induced E.M.F.; and currents will, therefore, flow in the rotor windings in a direction exactly opposite to that obtained under ordinary conditions of working. This *negative* rotor current necessarily implies a negative primary current; or, in other words, the component of the total primary current which balances the rotor current will now be forced to *flow back into the 'bus bars, against the primary impressed E.M.F.*, and the induction motor—when mechanically driven at a speed above synchronism—becomes a *generator of electric power.*

Such machines are known as asynchronous generators, to distinguish them from the more usual form of synchronous alternator or polyphase generator. It should be particularly noted that the periodicity of the currents obtained from the terminals of an asynchronous generator is *independent of the rotor speed*, being determined solely by the frequency of the E.M.F. supply brought to the terminals, for the purpose of producing the magnetic field, and without which these machines would not work.

(93) **Vector Diagrams of Asynchronous Generator.** — In Fig. 101 the vector diagram of an induction motor has been drawn to illustrate the effect of mechanically driving the rotor at a speed slightly above synchronism, and so transforming the machine into a generator of electric power. This diagram

should be compared with Fig. 92 on p. 254, and it will be seen how the difference lies in that the rotor current, C_R, has been drawn exactly equal to the current C_2 in Fig. 92, but *in an opposite direction.* The same assumptions have been made in order to simplify the diagram, these being (1) that the resistance of stator windings is negligible, and

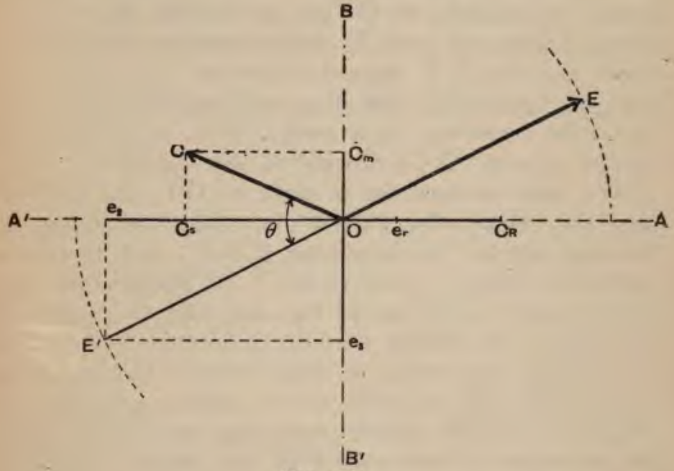

FIG. 101.

(2) that the rotor windings are without self-induction. This last assumption involves the idea of the *whole of the leakage magnetism* enclosing the *stator windings only*, and *not passing through the iron core on which the rotor conductors are wound.* The current vector, $\overline{O\,C_m}$, is, therefore, the magnetising component of that portion of the total magnetic flux which passes through both rotor and stator windings. It induces an E.M.F., e_2, in

the rotor coils; and, since these coils are being revolved at a *speed above synchronism*, the E.M.F., $\overline{e_2\, e_r}$, due to the rotation in the flux at right angles to the inducing flux, is now *greater* than the induced E.M.F., $\overline{O\, e_y}$, the result being a current, $\overline{O\, C_R}$, flowing *against* the induced E.M.F., and due to the effective or resultant E.M.F., $\overline{O\, e_r}$. The balancing component of the stator current must, of course, be $\overline{O\, C_S}$, exactly equal but opposite to $\overline{O\, C_R}$: the E.M.F. due to the leakage field will be $\overline{O\, e_3}$, which must now be drawn *below* the line A A′ instead of above, as in Fig. 92; and, by completing the diagram, we obtain $\overline{O\, E}$ as the total necessary impressed E.M.F. corresponding to the current, $\overline{O\, C_R}$, flowing in the rotor.

We now see how the product of $\overline{O\, E}$ and $\overline{O\, C}$ is *negative*, since the current in stator windings is flowing against the impressed E.M.F., and therefore returning power to the circuit. All the vectors are of the same length as in Fig. 92, and the angles θ are equal. It follows that the power given back to the circuit, according to Fig. 101, is the same as that which was supplied to the motor according to Fig. 92, and the power supplying the C^2 R losses in the rotor — represented by the product $O\, C_R \times O\, e_r$ — is taken from the driving source, and transmitted to the rotor through the shaft.

In Fig. 102 the asynchronous machine, A, and the synchronous machine, Z, are shown coupled together on a load which may consist of incandescent lamps or induction motors, or both. The connections are shown for one phase only, so as not to complicate the diagram. The current through the stator coils of machine A is C, corresponding to the vector $\overline{O\, C}$ in Fig. 101. This may be considered as made up of two components—the current, L, going to the load, and the current, S, passing through the armature windings of the synchronous machine, Z.

We shall suppose the condition of things to be as represented by Fig. 101—that is to say, the synchronous machine, Z, runs at a constant speed, and produces a definite pressure, E, at terminals, while the machine A runs at a constant speed somewhat above the speed of synchronism, which results in a definite value of the current $\overline{O\,C}$ and a definite phase angle θ between current and impressed E.M.F.

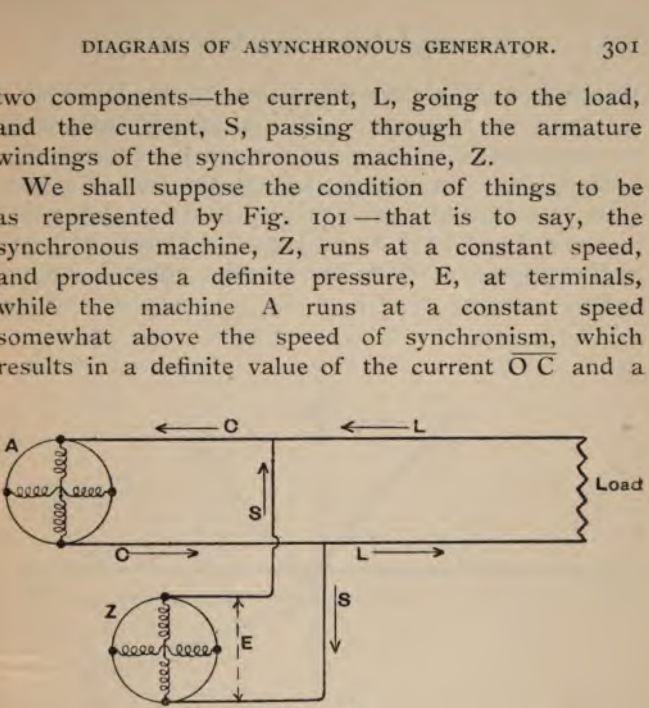

FIG. 102.

In Fig. 103 the diagram Fig. 101 has been re-drawn in order to show merely the current $\overline{O\,C}$ making an angle θ with the pressure vector, $\overline{O\,E'}$, which may be considered as the E.M.F. *at the terminals of the load*, while $\overline{O\,E}$ represents the E.M.F. generated by the synchronous machine. Drop a perpendicular, $C\,L_1$, on to $O\,E'$, and note that, *if the load is non-inductive* and equal to the product $O\,E' \times O\,C \cos\theta$, then the asynchronous machine, A, is doing the whole of the load, while the synchronous machine, Z, acts merely the part of an exciter, and does no work, since the current component, O S_1, is "wattless." If the load is greater than this amount, and equal to O E′ × O L_2, then machine A will still do its share, and give the whole of its output to the load; but the balance required (which is equal to O E × L_2 L_1) must be supplied by the machine Z, the total output of which may be written (O E) × (L_2 C) cos β, where L_2 C is the current S in the diagram Fig. 102.

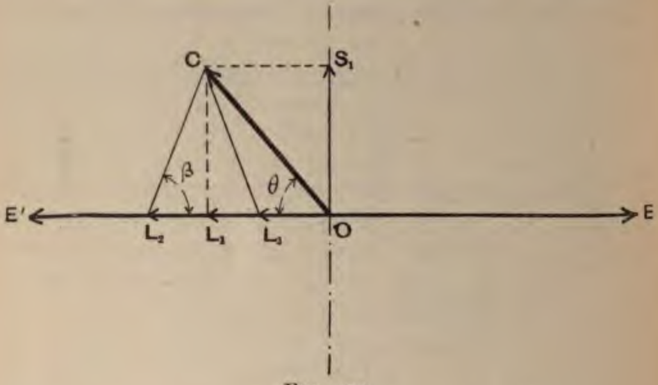

FIG. 103.

If the external load is *less* than the full output of the asynchronous machine, as, for instance, O E′ × O L_3, then the current, S, in the armature of the synchronous machine is $\overline{L_3\ C}$, which makes an angle with O E greater than 90 deg. It follows that the power given out by machine Z is negative—*i.e.*, this machine is being *driven as a motor* by the machine A, the power expended in doing this being equal to O E′ × L_3 L_1.

Let us now see what is the effect of an inductive load. The diagram Fig. 104 has been drawn in a similar manner to Fig. 103, but the load current now lags behind the pressure $\overline{\mathrm{O\,E'}}$, by a certain angle ϕ. If the load current is $\overline{\mathrm{O\,L_1}}$, such that C L_1 is at right angles to O E′, then the *power* absorbed is the same as before, being equal to O E′ × O P or to O E′ × O C cos θ, which is the total output of the asynchronous machine — for the particular conditions as regards speed and excitation that we have assumed. But the "wattless" current, $\overline{\mathrm{O\,S_1}}$ from the armature of the synchronous machine is now greater than $\overline{\mathrm{O\,S_1}}$ in Fig. 103 by an amount P L_1, which is exactly equal to the wattless component of the load current; from which it is

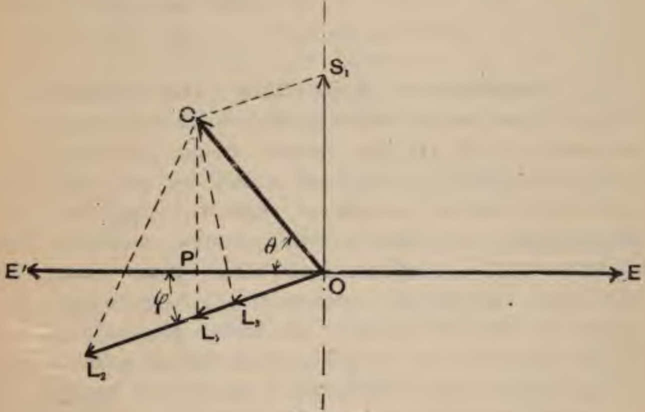

FIG. 104.

clear that any "wattless" currents required, owing to the load having self-induction, cannot be provided by the asynchronous machine, but must be supplied by the synchronous generator *in addition to the wattless exciting current* required by the former. If $\overline{O\,L_2}$ is the load current, $\overline{L_2\,C}$ is the current S (Fig. 102) from machine Z; while, if the load current is $\overline{O\,L_3}$, the power absorbed is less than the full output of the machine A, and the synchronous machine is again being driven as a motor.

(94) **Conclusions Regarding the Induction Motor used as a Generator.**—Since the frequency is independent of the speed of an asynchronous generator (being determined solely by the speed of the synchronous machine which supplies the magnetising currents to the stator winding), it follows that machines of this class can be paralleled without the necessity of regulating the speed to such a nicety as when paralleling the more common type of alternating-current generators. In fact, the various units have merely to be run up to approximately synchronous speed, and switched on to the 'bus bars, much in the same manner as when coupling direct-current dynamos. The proportion of the total load taken up by each machine (where several are coupled in parallel), will depend solely upon the relative *speeds* at which the various units are driven.

It would almost seem as if this convenience in paralleling were the only recommendation for machines of this class.

They have the disadvantage of requiring a synchronous generator, always running, to fix the frequency and provide the wattless magnetising currents; moreover—since the asynchronous machines are only capable of supplying the external circuit in the form of *energy* currents — the synchronous generator may have to be of a very large size if the load is inductive, seeing that it will be called upon to supply the wattless component of the total load current, in addition to the magnetising currents of the other generators.

With the advent of the compensated induction motor as introduced and developed by Mr. Alex. Heyland and others, there is a possibility of such machines — when used as generators — proving a commercial success, owing mainly to the fact that they can be loaded inductively, thus bringing the size of the synchronous exciter within reasonable limits. It is not proposed to discuss this question at greater length; but since there is a possibility of the compensated polyphase induction motor with commutator being used commercially in the near future, the theory and properties of such motors will be briefly dealt with in the following article.

There is one point in connection with the asynchronous generator which should, perhaps, be mentioned, and this is the peculiar effect of a heavy overload. If the speed of the prime mover be increased beyond a certain limit, the proportion of the load taken by the generator *will begin to fall off*, and unless suitable precautions are taken, there is a possibility of the engine "running away." This will be evident from a study of Fig. 101, which, as already pointed out, differs but little from the corresponding diagram of the same machine used as a motor. In the case of the motor it has been shown how, when the torque exceeds a certain limit, the leakage field becomes so great that the machine is unable to respond to the additional call upon it, and falls out of step. In the generator the effect is similar, and the overload capacity of the machine is reached when the speed exceeds the speed of synchronism by an amount approximately equal to the "slip" corresponding to the maximum output of the same machine when run as a motor.

(95) **"Compensated" Polyphase Motors with Commutators.** — About five years ago Mr. Alex. Heyland had the idea of supplying the magnetising currents of an induction motor through the rotor windings instead of through the stator coils. This ingenious method has the effect of improving the power factor, which can thus be brought up to about 100 per cent. for all loads. Other workers, including Messrs. Gorges, Latour, and Osnos, have devised machines somewhat on the same lines, but since there is no difference in the principle involved, we shall content ourselves with briefly considering the reason why, by feeding the magnetising currents through a commutator into the rotor windings, it is possible to compensate for phase displacement, and bring the power factor, even at light loads, approximately up to unity.

In Fig. 105 the rotor is supposed to be of the wound type, and provided with a commutator in all respects similar to that of a direct-current bipolar machine. Imagine the rotating field to be produced by two alternating fields, one acting in the direction B B′, and the other—differing in phase by a quarter period — acting in the direction A A′, at right angles to B B′; this being the method of treating the subject which was justified and explained in the last chapter, when describing the action of the ordinary induction motor.

Consider, first, what goes on in the phase B B′ only. The stator windings, in this phase, act merely

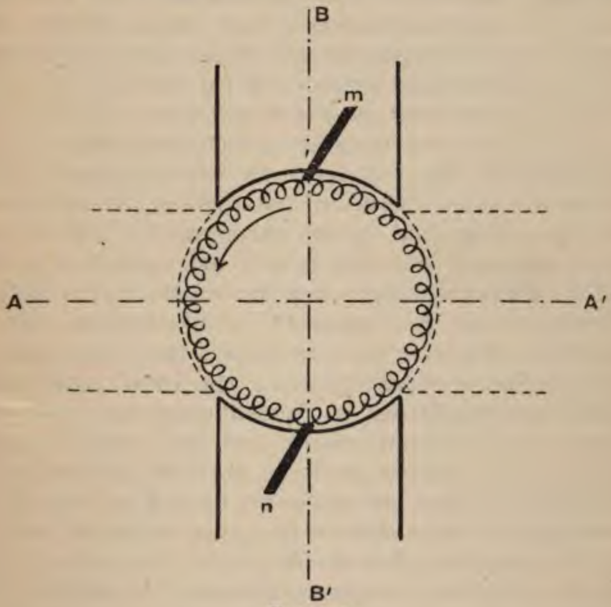

FIG. 105.

as the primary of a static transformer; and, if we suppose the rotor or secondary windings to be without any short-circuiting connections, the current that will flow in the stator windings when these are connected to the supply mains will be the magnetising current; and although the *true* magnetising watts—*i.e.*, the product of the current and *resultant* volts—may be small, the *apparent* watts are relatively great, since they are equal to the product of the current and the full pressure of the supply.

Suppose now that, with the armature at rest, and with the stator windings disconnected from the supply, an alternating current of the same frequency, but suitably reduced pressure, is fed into the brushes, *m* and *n*, bearing on the commutator at the two ends of a diameter lying on the magnetic axis, B B′, as shown in Fig. 105. The current will divide itself between the two halves of the winding, and produce an alternating field in the direction B B′, which—if the pressure be adjusted to suit the number of turns in the rotor winding—may be made of the same strength as the field originally produced by the stator winding. But *with the rotor at rest* the power factor has not been improved, since the rotor coils may now be considered as constituting the primary winding of a transformer, and the same trouble occurs—*i.e.*, the magnetising current produces the the magnetic flux, which, in its turn, gives rise to a back E.M.F. of self-induction very much in excess of the resultant E.M.F. required to overcome the resistance of the windings, and this necessitates an impressed E.M.F. slightly greater than the induced E.M.F. and nearly 90 deg. in advance of the current, as regards phase.

Now imagine the rotor to be revolving at synchronous speed. The brushes bear on the commutator, but retain a fixed position in space, the result being that the alternating current fed into the brushes will produce an alternating flux in the same direction, and of the same frequency as before; *this frequency being quite independent of the speed at which the rotor is revolving,* and depending merely upon the frequency of the supply connected across the brushes. It is true that the frequency of the current in *any particular portion of the rotor winding* will depend upon the speed of revolution, and, in fact, for the condition of synchronous speed that we are considering, the current in the rotor conductors would be a continuous one; but the point which concerns us at present is that, as the rotor revolves, the conductors lying between the brushes, *m* and *n*, are cutting the field A A′ at such a rate as to produce an alternating E.M.F. of rotation exactly equal, but opposite in direction, to the alternating E.M.F. of induction due to the field B B′. No further difficulty need therefore be experienced in getting the magnetising current through the rotor windings. The E.M.F. across the brushes, *m* and *n*, no longer requires to be of an abnormally great value, since the induced back E.M.F. is now balanced by the E.M.F. of rotation. A few volts only are needed to overcome the ohmic resistance of the windings, and provide the necessary magnetising current. Moreover, this pressure will now be *in phase with the magnetising current*, and not nearly 90 deg. in advance, as was necessary so long as the back E.M.F. of self-induction was a factor to be dealt with. It follows that the magnetising watts are now *true watts*, and, by regulating the pressure across the brushes, *m* and *n*, until the magnetising ampere turns are equal to those which otherwise would have to be provided by the stator current, the latter may be reduced to nearly zero, thus bringing the power factor (even with the machine running light) to a value very near to unity.

When the motor is loaded, the magnetising current requires to be of approximately the same value as on open circuit, and the power factor can be maintained at nearly 100 per cent. for all loads. It is evident that, as the load comes on the motor, the speed is no longer that of synchronism; but it does not differ much from the speed of the rotating field, and at full load, with a "slip" of, say, 3 per cent., the currents in the rotor conductors, although no longer direct currents, are of such a low periodicity that self-induction troubles do not arise.

This is the theory of the compensated induction motor; it will be understood that the exact pressure across the brushes should be adjusted experimentally for each particular machine, to suit the supply voltage. If the pressure is too low, the exciting current in the rotor windings will be insufficient to provide the full amount of the flux, and it will, therefore, be supplemented by a small magnetising component of the stator current, which means that the power factor will still be less than unity. If too great a current is put into the rotor windings, this will give rise to a *leading* component of the primary current of such a value as to neutralise the excess of magnetising current in the rotor, and maintain the resultant magnetising ampere turns at the exact value required to produce the necessary back E.M.F. in the stator windings.

Although it is not proposed to enter into details of construction, or to discuss the various devices by means of which this theory of compensation may be applied in practice, this article would not be complete without a reference to the actual method of compensating induction motors originally devised by Mr. Heyland.

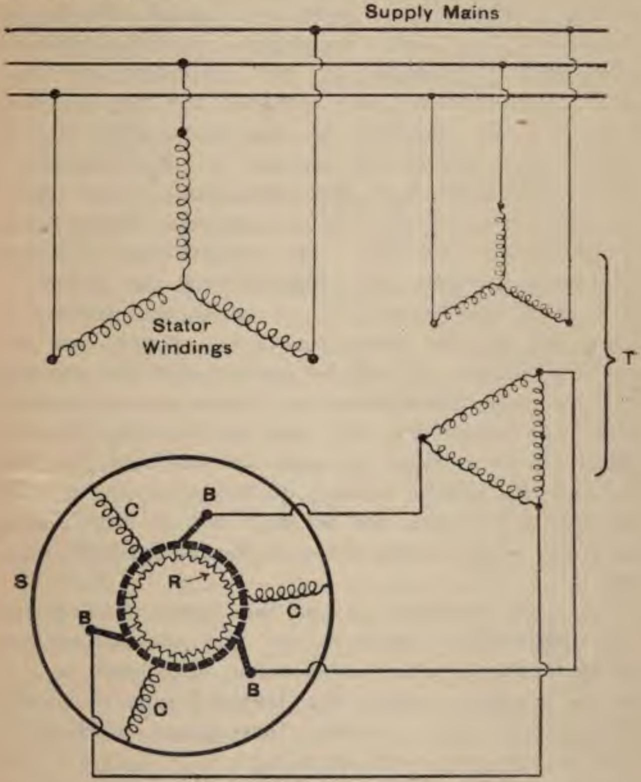

FIG. 106.

In Fig. 106, the stator windings of the motor are shown star-connected off the supply mains. The three rotor coils C C C, are also star-connected, being short-circuited at their outer ends by the heavy connection, S, while their inner ends are joined through the *low resistance*, R. This resistance actually takes the form of small V-shaped connectors of low-resistance material joining contiguous segments of the commutator; and the currents that are brought to the brushes, B B B, will circulate in the rotor coils, C, in such a direction as to produce a rotary magnetic field synchronous with the field that would otherwise be produced by the magnetising components of the stator currents. The magnetising currents led into the brushes are obtained from the secondary of a small transformer, T, of which the primary is connected to the same source of supply as the stator windings. It will be noticed that the primary coils of this transformer are shown star-connected, while the secondary coils are mesh-connected; the object of this being to provide currents to the brushes, B, which, instead of being in phase with the supply pressure, are 90 deg. out of phase, and, therefore, of the same phase as the required magnetic flux.

It is not necessary to use the same winding for the magnetising currents as for the circulation of the currents which arise when the load is put on the machine, and in the Heyland type of motor of recent design a double three-phase winding is used on the rotor with advantage.

Such machines when used as asynchronous generators have undoubted advantages over the ordinary type of machine, and it will be readily understood that there are no difficulties in the way of compounding these compensated generators, since the necessary currents for strengthening the field can be taken off suitable current transformers, and led into the commutator through brushes in the same manner as the compensating currents.

Although the compensated and compounded polyphase induction motor is undoubtedly interesting and instructive from the scientific point of view, it is doubtful whether it has come to stay, because unless the advantage of an improved power factor is worth the increased complication and expense of manufacture, it can never prove a commercial success.

It must not be overlooked that the addition of a commutator, with all the increased difficulties of winding, is out of the question for small machines; and when we have to deal with large machines, the power factor of these at full load is already nearly 90 per cent., and in order to raise this to 100 per cent., it is still a doubtful point whether the increased cost and liability to break down are justified.

(96) **Rotary Converters.**—The rotary converter is a machine similar to the double-current generator described in article 36, Chapter III. It is generally used for transforming polyphase into direct currents, the polyphase currents being led into the armature windings through slip rings, while the direct current is collected by brushes bearing on the commutator. The machine may, therefore, be considered as a combined synchronous motor and direct-current generator.

When treating of polyphase generators, it was shown how the sides of a closed polygon of vectors represent the respective alternating E.M.F.'s in the various (mesh-connected) armature coils. Thus, in Fig. 107(*a*), the vectors $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, forming the sides of an equilateral triangle, represent, by their magnitude and direction, the amount and phase relations of the E.M.F.'s in the three windings of a delta-connected three-phase generator or motor.

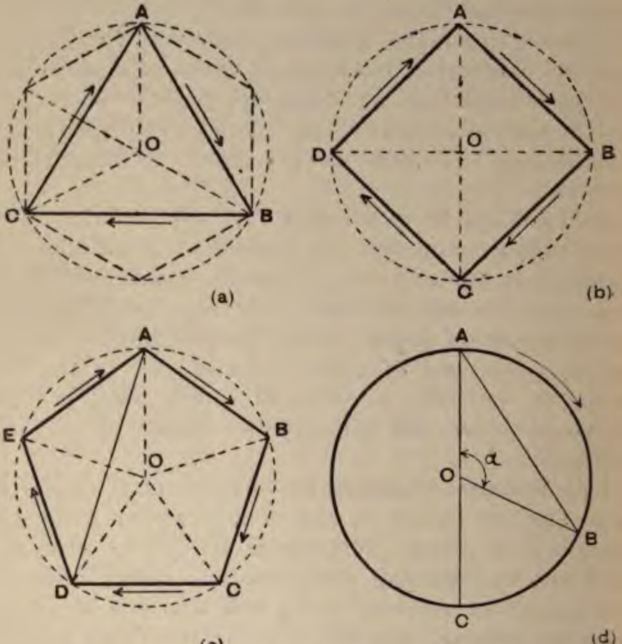

FIG. 107.

In Figs. 107(*b*) and 107(*c*), the vector diagrams have been drawn for four and five phase machines *respectively*, while the dotted hexagon in Fig. 107(*a*) refers to a six-phase machine. The fact that all these figures are *closed* polygons is an indication that the sum of all E.M.F.'s acting in the armature winding is zero, and consequently currents will not circulate in the coils when the external circuit is open.

Now, since $\overline{\mathrm{A\ B}}$, $\overline{\mathrm{B\ C}}$, etc., represent the alternating E.M.F.'s in the various sections of the winding, which are all connected in series, the pressure between any two terminals of the polyphase circuit (which is equal to the sum of the component E.M.F.'s) will be indicated on the vector diagrams, both in magnitude and phase, by the straight line joining the beginning of the first and the end of the last vector of which the sum is taken. Thus, in Fig. 107(*b*), the pressure between opposite terminals is indicated by the lines A C and B D, the length of which is a measure of the E.M.F. obtained by connecting two of the phases in series: the *phase* of the resultant E.M.F. is also indicated by the angular position of these lines relatively to the component vectors; but the direction—*i.e.*, whether, for instance, from A to C or C to A—cannot be shown by an arrow on the diagram, as it will depend upon what is to be considered as the positive direction in the external circuit joining the two terminals.

In Fig. 107(*c*), the pressure between the terminals A and D will be represented by the length of the line A D, which may be looked upon as the sum of the three vectors $\overline{\mathrm{A\ B}}$, $\overline{\mathrm{B\ C}}$, and $\overline{\mathrm{C\ D}}$, or of the two vectors $\overline{\mathrm{D\ E}}$ and $\overline{\mathrm{E\ A}}$, for these must necessarily balance, and give the same resultant.

Let us now carry this construction further, by increasing the number of the sides of the polygon, and therefore the number of phases, indefinitely. This leads us to the diagram Fig. 107(*d*), which represents the E.M.F. in the armature winding of the machine as being due to the rotation, in the field, of an infinite number of armature sections, in each of which an alternating E.M.F. is generated. The circle may be considered as the limit of the polygon of vectors as the sides of the polygon are increased in number without limit; and the length of the line A B represents the alternating pressure obtained between two tappings from the winding separated by a distance equivalent to the phase angle α. With regard to the diameter A C, this represents the pressure obtained between two diametrically opposed points of the (two-pole) armature winding; and if we suppose the E.M.F. wave to follow the simple harmonic law of variation, the direct-current pressure between the brushes bearing on the commutator of a rotary converter or double-current generator would be $\sqrt{2}$ times the value given by the length A C.*

The diagrams, as drawn in Fig. 107, are not limited in their application to two-pole machines; it is merely necessary to bear in mind that the angles in the diagrams are *phase angles*, and the vector A C in Fig. 107 (*b*) or (*d*) indicates the pressure between two points on the winding separated by a distance equal to the pitch of the poles: thus, it is only in the case of a two-pole machine that these tappings would actually be at the two ends of a diameter.

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* If the E.M.F. vectors such as $\overline{AB}$ and $\overline{AC}$ stand for the *maximum* values of the alternating pressures instead of the $\sqrt{\text{mean square}}$ values, then the diameter of the circle would correctly indicate the direct-current voltage between brushes, no multiplier being necessary. It is evident that, when tappings are taken from diametrically opposed points on the winding, these points must simultaneously pass under the direct-current collecting brushes twice during each revolution of the armature, and at these moments the instantaneous value of the alternating E.M.F. must necessarily correspond with the direct-current voltage: moreover, since the brushes lie on the neutral axis, where the E.M.F. is at its maximum, it follows that it is the *maximum* value of the alternating E.M.F. which agrees with the direct-current voltage.

A rotary converter can evidently not be wound for any desired pressure transformation; the ratio of alternating to direct current pressure is fixed, being determined solely by the number of slip rings or phases. From an inspection of the diagrams in Fig. 107, it will be seen that, since the length of the chord joining two points on the circle is a measure of the alternating E.M.F. between these points, the maximum value of this E.M.F. is given by the expression

$$E_{\text{max.}} = 2\,R \sin \frac{a}{2},$$

where R is the radius of the circle and a is the phase angle between the two tappings off the armature winding. Expressed in terms of the direct-current voltage, we have

$$E_{\text{max.}} = D \sin \frac{a}{2},$$

where D—the diameter of the circle—stands for the direct-current pressure across brushes. If we assume the sine curve wave form, the $\sqrt{\text{mean square}}$ value of the alternating pressure will be equal to the above value divided by $\sqrt{2}$.

The following table gives the pressure per phase for various numbers of slip rings, the direct-current pressure being taken as unity :

Number of phases or slip rings.	Sin $\frac{a}{2}$	E.M.F. between slip rings = $\sin \frac{a}{2} \div \sqrt{2}$
Single phase or two phase	1	·7071
Three phase	$\frac{\sqrt{3}}{2}$ = ·866	·6124
Four phase	$\frac{1}{\sqrt{2}}$ = ·7071	·5000
Five phase	·5878	·4157
Six phase	½ = ·5000	·3536
Twelve phase	·2588	·1830

These calculated values agree fairly well with figures obtained from actual machines. For instance, the voltmeter readings taken from a couple of American machines were 177 for a two-phase converter, and 155 for a three-phase machine ; the direct-current voltage, in each case, being 250. The calculated pressures, on the assumption of the sine curve wave form, are

$$250 \times \frac{1}{\sqrt{2}} = 177 \text{ (nearly)}$$

for the two-phase machine, and

$$250 \times \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} = 153$$

for the three-phase machine, which agree well with the measured volts.

(97) **Elementary Theory of the Rotary Converter.**—Consider a rotary converter running at full speed with the direct-current circuit open — *i.e.*, without any direct current being taken out of the armature. The condition is merely that of a synchronous alternating-current or polyphase motor running light. Let us suppose the field coils to be excited from an independent direct-current source, and note that—when the armature is revolving at synchronous speed—there is a definite strength of field required to produce the necessary back E.M.F. of rotation in the armature conductors. If the current in the field coils is adjusted to give this particular strength of field, the alternating currents fed into the armature through the slip rings from the supply mains will be very small, being merely such as to produce the torque required to overcome the light load losses; moreover, this current will be in phase with the applied E.M.F.

If we now either increase or decrease the field current, the result will be an alteration in the armature current, which must adjust itself in order that the resultant magnetising ampere turns may be the same as before: thus, if the current in the field coils be strengthened, the armature current will *lead* the applied E.M.F., while, if the excitation is reduced, the current in the armature will be a lagging one.[*]

Once the idea of a constant resultant field excitation is clearly understood, it is an easy matter to follow the actions which take place when a direct current is taken out of the armature through the brushes bearing on the commutator. Whatever portion of the direct current passes through sections of the armature winding, must be balanced at every instant by currents taken from the polyphase supply mains, in order that the resultant field excitation shall not be altered. It is true that the direct current taken from the armature tends mainly, if not wholly, to produce cross magnetising ampere turns; but these cross magnetising turns must nevertheless be balanced by the polyphase current entering the armature through the slip rings, because —even on the assumption of an efficiency of 100 per cent.—whatever power is taken out on the direct-current side, must evidently be balanced by an equal power supplied to the armature from the polyphase side.

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[*] Incidentally, attention may be called to the possibility of regulating the power factor of a polyphase system by over or under exciting the field coils of rotary converters connected to the mains. These machines will, in this respect, act in a similar manner to synchronous motors, and they may be made to produce the effect of either condensers or choking coils as the case may require.

On account of this balancing of the magnetising effects due to the continuous and polyphase currents, there is practically no armature reaction in the polyphase rotary converter. If the losses are negligible, and the field current is adjusted to give the maximum power factor, there is theoretically very little or no armature reaction, and this accounts for the fact that it is not necessary to alter the position of the brushes with varying load. In this respect the rotary converter differs from the double-current generator, because, in the latter, both direct and alternating currents combine to produce field distortion (see article 36, p. 110).

It is well known that, in the continuous-current transformer with two windings and two commutators, there is no trouble with sparkling at the brushes, owing to the fact that the cross magnetising ampere turns of the secondary winding are balanced by the ampere turns of the primary winding ; and although, in the polyphase rotary converter, there is only one winding, and the current taken out of the machine is not of the same kind as the current put in, yet, at every instant, there is a balancing action similar to that which occurs in the continuous-current transformer.

At certain moments during the revolution of the armature, the current will pass direct from slip ring to commutator brush, and at all other times the transfer of energy may be considered as due either to direct conduction through the windings between slip rings and brushes, or to balanced generator and motor action—certain portions of the winding acting as generator while the other portions act as motor. There is an exception in the case of the single-phase rotary converter, the action of which is slightly more complicated ; here some of the energy (about one-third of the total) must be considered as transferred by *unbalanced* generator and motor action, which involves the idea of a flywheel effect, and a certain storage of energy in the revolving armature during portions of a complete period, this energy being given back again in the form of direct currents at other instants. This leads, in some cases, to sparking, and it is one of the reasons why the single-phase rotary converter is not a very satisfactory machine.

(98) **Output and Efficiency of Rotary Converters.** — When dealing with double-current generators (article 36, p. 110), the question of

C^2 R losses in the armature windings was discussed, and it is not proposed to make a thorough investigation of this matter in connection with rotary converters. It will be sufficient to point out that the output of these machines—*except in the case of the single-phase converter*—is always greater than that of the same machine used merely as a direct-current generator; that is to say, the C^2 R losses are considerably *less* than if the whole of the output were passed through the windings in the form of a direct current. That this must be the case is fairly obvious, especially when it is remembered that, as the connections from the slip rings pass under the brushes, there is a direct transfer of current from slip ring to brush *which does not pass through the armature conductors.*

The output of a rotary converter will also be increased with the number of phases or slip rings. Thus, the C^2 R losses in a machine provided with three slip rings, and fed by a three-phase current, will be greater than if the same machine is provided with six slip rings and fed by a six-phase current. This, indeed, is what would be expected, and one reason is that the increased number of phases reduces the current *per phase* required to balance the continuous current. For instance, if we assume a constant direct-current output of 100 kw. at 100 volts and a power factor of unity on the polyphase side, the current per phase in each armature section for a total input of 100 kw. (all losses neglected) would be 544 amperes for a three-phase supply, and only 472 amperes for a six-phase supply.

From figures given by Messrs. Steinmetz and

Mershon, the following table has been prepared. It gives the approximate output of a rotary converter with different numbers of slip rings, the output of the same machine as a direct-current generator being taken as unity. The power factor is assumed to be 100 per cent.

Number of slip rings or phases.	Output of rotary converter.
Two (single phase)	·85
Three	1·35
Four (two or four phase)	1·65
Five	1·8
Six	1·95
Eight	2·05
Ten	2·15
Infinity	2·3

If the power factor is less than unity, the output will be less; thus, if $\cos \theta = {\cdot}8$ the output of the single-phase converter would be only ·7 while that of the three-phase and four phase machines would be about 1·15 and 1·4 respectively.

Another advantage of increasing the number of phases beyond three is that the heating of the armature conductors is more evenly distributed. There will always be a greater amount of heat generated in the neighbourhood of the points on the winding from which the connections are taken to the slip rings; and if the number of these points is increased, it stands to reason that there will be less local heating and that the $C^2 R$ losses will be dissipated more uniformly throughout the entire armature.

Since the output of a rotary converter increases with the number of slip rings or phases on the alternating-current side, it is obvious that, if an *economical* and simple means can be employed for

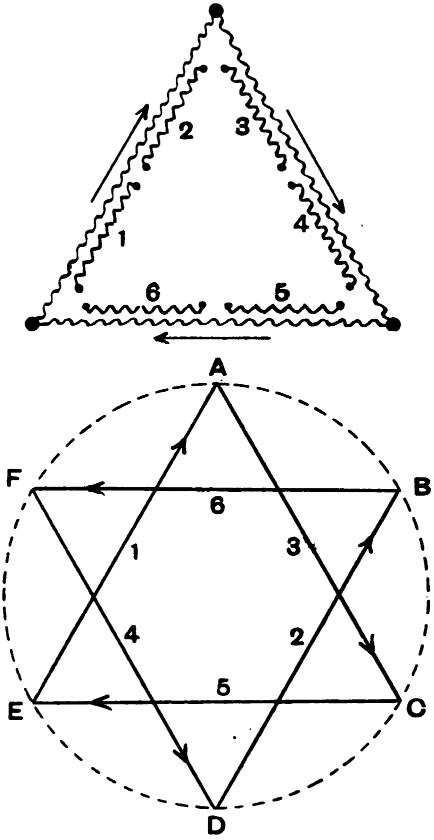

FIG. 108.

transforming a two or three phase supply into a polyphase system having a larger number of phases, this may be advantageous under certain conditions. The problem, indeed, is not a difficult one, as many solutions can be obtained by a suitable combination of vectors differing in phase. Fig. 108 illustrates one method (described by Mr. A. C. Eborall in his admirable paper on polyphase sub-station machinery*) by means of which a three-phase supply can readily be transformed into a six-phase system suitable for feeding a rotary converter provided with six slip rings.

The upper diagram shows the primaries of the three step - down transformers mesh - connected between the supply mains. Each transformer is wound with two equal but distinct secondaries, and the six sections of secondary winding thus obtained are connected up in such a manner as to produce a six-phase supply. The combination of vectors is shown in the lower diagram, and it will be seen that the desired result is obtained by mesh-connecting the coils 1, 3, and 5 in one direction, and the coils 2, 6, and 4 in the opposite direction. By feeding the slip rings from the junctions A, B, C, D, E and F, the six-phase supply to the armature is obtained.

Messrs. C. P. Steinmetz and L. R. Emmet have devised an equivalent arrangement of static transformers, very much on the lines of Mr. Scott's phase-transforming system, by means of which the same end is attained.†

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* "Some Notes on Polyphase Sub-station Machinery," by A. C. Eborall, read before the Institution of Electrical Engineers, March 14, 1901 (vol. xxx, p. 702, *Journal*, Inst.E.E.).

† See *Western Electrician* (Chicago), Oct. 25, 1902.

Mr. A. D. Lunt has shown how a twelve-phase system can be obtained in a similar manner from a three-phase supply, but the supposed advantages of increasing the number of phases to this extent are not obvious. Even in the case of very large rotaries, the disadvantages of having twelve slip rings, with all the increased complication and expense, would seem to outweigh the possible advantage of slightly improved efficiency or increased output. The theoretical output of the twelve-phase rotary is only about 12 per cent. greater than that of a six-phase machine.

The efficiency of rotary converters compares favourably with that of motor-generator sets: it would be about 90 per cent., at full load, for a three-phase 200-kw. machine at a periodicity of 40, or for a 400-kw. machine at a periodicity of 60, while the efficiency of an equivalent motor-generator set might be about 86 per cent. It should be understood that these figures include the losses in the step-down transformers required for use with the rotary converter; but the motor of the motor-generator set would be wound for the high-tension supply, which might be of the order of 5,000 volts.

Tests have been made, and figures published, tending to show that, as regards the all-day efficiency of rotaries and motor-generators, there is not much to choose between the two; but the cost of the motor-generator is generally somewhat higher than that of the equivalent rotary converter, although, when the necessary step-down transformers and regulators are taken into account, the difference in this respect is not so great as is sometimes supposed.

(99) **Starting and Synchronising Rotary Converters.**—When possible, it is desirable to start up a rotary converter from the direct-current side: this is a simple matter, and the operation is in all respects similar to the method adopted for starting up an ordinary shunt-wound direct-current motor. The machine is then synchronised by varying the field excitation until the correct speed—as indicated by the synchroniser—is obtained, when the switches on the polyphase side can be closed. If the supply is three-phase, it is usual to have one of the switches closed, and, at the moment when synchronism is attained, the other two switches are thrown in. The operation of synchronising has to be carefully carried out, or trouble may ensue. In cases of emergency, the process of synchronising may be omitted; the machine should be run up to a speed slightly above synchronism, when the *direct-current switches must be opened*, and the switches on the polyphase side immediately closed, thus connecting the armature to the polyphase supply. The machine will pull itself into step, and the direct-current switches can then again be closed.

When it is not possible to start up from the direct-current side, a very satisfactory method consists in providing a small induction motor for the purpose of running the armature up to speed. This auxiliary motor is designed to run at a slightly higher speed than the rotary, and the latter is then brought down to synchronous speed by loading one of the phases through a suitable rheostat. This method makes the process of synchronising exceedingly simple, and, as the power developed by the induction motor need only be about one-tenth of the full load output of the rotary, the extra cost is not very great.

There is another method of starting up from the polyphase side which, however, is only applicable to small machines, and this consists in switching the polyphase supply directly on to the machine, having previously taken the precaution to disconnect the field windings and divide them into several sections: the main switches on the direct-current side must, of course, also be open. The revolving field due to the polyphase currents in the armature, causes the machine to start up much in the same way as an induction motor; and when synchronism is reached—as indicated by the steady reading of a voltmeter across the direct-current brushes—the field circuit may be closed; but certain precautions must be taken to ensure that the field circuit is closed on the phase which will give the right polarity across the direct-current brushes. The object of dividing up the field winding into several distinct sections is to prevent the accumulated induced E.M.F. in these windings, during the period of running up, reaching such a high value as to risk a breakdown of insulation.

The chief objection to this method of starting is that a very large current is taken from the supply mains, amounting to, perhaps, three times the normal full-load current of the rotary, and since this current is very much out of phase with the supply pressure, the regulation of the polyphase system will be seriously affected. It is true that, before switching on the supply, provision can be made for reducing the voltage across slip rings by taking intermediate *tappings* from the low-tension side of the step-down transformers, but the extra cables and switch gear required are a somewhat serious objection.

(100) **Regulation of Rotary Converters.**—Since the direct-current pressure across brushes has a definite value for a given alternating pressure across slip rings, it follows that the regulation cannot be effected, as in the case of a motor-generator, by merely altering the strength of the field current. Any alteration in the current passing through the field windings is immediately met by an alteration in the armature current, which so adjusts itself as to leave the resultant field as before—*i.e.*, of such a value as to produce the required back E.M.F. corresponding to the speed of synchronism.

If the pressure of the polyphase supply is raised or lowered, then the direct-current pressure will be varied in a corresponding manner. This suggests what is, perhaps, the most perfect method of regulation — namely, the employment of variable ratio step-down transformers, by means of which—even with constant pressure at the high-tension terminals of the polyphase supply—the volts across slip rings can be varied within the required limits. This regulation can be done by hand or automatically; but automatic regulation on these lines would be costly and liable to give trouble owing to complication of parts.

Another method of varying the pressure across slip rings, which is automatic in its action, is of considerable interest, and since it is largely used in practice, it is advisable that the principles involved be clearly understood. In the first place, this method requires the compound winding of the field magnets; but, from what has already been stated, it is evident that the mere strengthening of the field ampere turns as the load increases, will not produce the required pressure rise across slip rings to compensate for the volts lost in overcoming

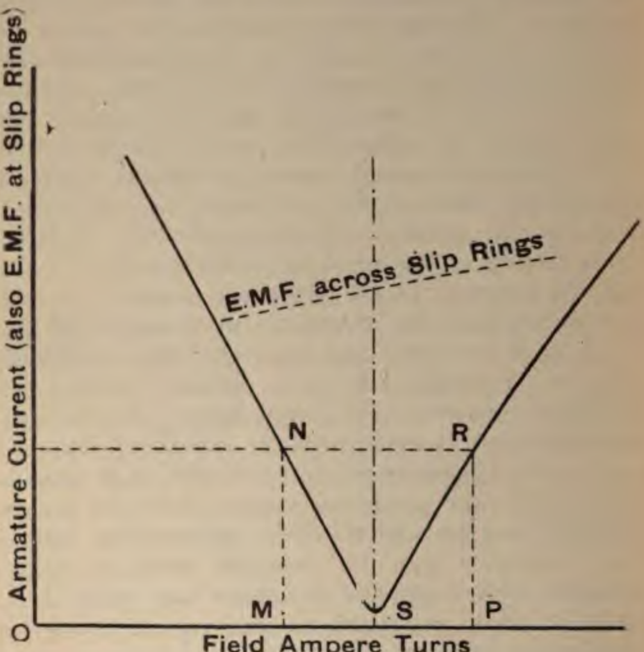

FIG. 109.

the resistance of the windings, *except under special conditions* which permit of this result being attained. These conditions are that there must be an appreciable amount of self-induction in the circuit between the high-tension supply terminals (at constant pressure) and the slip rings of the rotary, and that the ohmic resistance of this portion of the circuit must be relatively small. If the step-down transformers are of a type having a fair amount of magnetic leakage, it may not be necessary to provide additional self-induction in the form of choking coils; but, in practice, small choking coils in each of the polyphase leads are usually inserted.

In Fig. 109 the full line curve shows the relation between armature current and field excitation for a rotary converter or synchronous motor running light. The ordinates, or vertical measurements, indicate the amount of the armature current, while the horizontal measurements represent the ampere turns on the field magnets. There is a certain value of the field current—denoted on the diagram by the letter S—for which the armature current is a minimum; the power factor is then approximately unity, and the very small armature current passing into the windings is merely what is necessary to run the machine light. If now, we imagine the field current to be reduced to a value indicated by the distance O M in Fig. 109, the armature current will immediately rise to the corresponding value M N, and this will be *a lagging* current, the principal component of which is a magnetising current 90 deg. behind the impressed E.M.F., because this will produce a field flux in the same direction as the direct current in the magnet coils, and its value will be such as to provide the magnetising ampere turns which have been taken off the field coils.

If, on the other hand, we strengthen the field current, the main component of the armature current will *lead* the impressed E.M.F. by 90 deg., and so counteract the magnetising force of the additional field ampere turns.

For a given value, O P, of the field current, the no-load armature current may be of exactly the same value as when the field current was O M; but in the first case it will be *in advance* of the impressed E.M.F. by nearly a quarter period; while in the latter case, it will *lag behind* the impressed E.M.F. by the same phase angle.

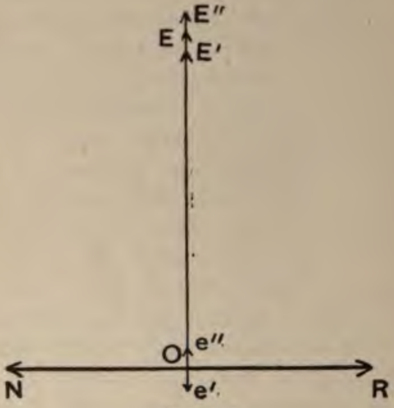

FIG. 110.

Now consider the vector diagram Fig. 110. Here $\overline{OE}$ represents one phase of the impressed polyphase E.M.F., which is supposed to be of constant value. We shall not complicate the diagram by taking into account the step-down transformers, and the length O E must be considered as representing the primary pressure *reduced to the pressure at slip rings*, in accordance with the transformer ratios, for the condition of the correct field excitation—*i.e.*, when the field current is such as to make the power factor a maximum (O S in Fig. 109). Under this condition the armature current is practically zero. Let us now see what will be the pressure at slip rings with a reduced field excitation and, therefore, a lagging armature current, which may be represented by the vector $\overline{\text{O N}}$.*

Note, first, that, *if there were no self-induction in the circuit*, the pressure across slip rings would remain unaltered (except for the small resistance drop, which may be neglected); but if the circuit has self-induction, the back E.M.F. due to the current $\overline{\text{O N}}$ will be $\overline{\text{O } e'}$, drawn exactly 90 deg. behind $\overline{\text{O N}}$. The result will be that the pressure at slip rings will be reduced to $\overline{\text{O E}'}$, which is less than O E by the amount O e'. Suppose now that the field current is increased until the armature current is a leading one of value O R; then $\overline{\text{O } e''}$ will be the induced E.M.F. due to this current, and it will be in phase with $\overline{\text{O E}}$: the pressure at slip rings will, therefore, be $\overline{\text{O E}''}$, which is equal to O E + O e''.

We are now in a position to plot the dotted curve in Fig. 109, showing how the pressure at slip rings increases with increased field ampere turns, and from this it is easy to understand that a rotary converter provided with a shunt winding across the brushes, and a series winding carrying the load current, may—by suitably adjusting the amount of self-induction—be made practically self-regulating at all loads.

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* In this diagram (Fig. 110) the current vectors are drawn exactly 90 deg. out of phase with the pressure vector, although in practice this condition is never quite reached. But, with the machine running light, the phase angle will be very nearly 90 deg., and, in any case, even if the machine were loaded, it would be correct, for the purpose of this argument, to consider $\overline{\text{O N}}$ as the magnetising component of the total armature current, because the "work" component, in phase with $\overline{\text{O E}}$, will produce only cross magnetising effects, and, even in the choking coils, will not give rise to a back E.M.F. having any appreciable effect on the regulation.

It is important to bear in mind that, in order to obtain this result, the ohmic resistance of the cables, transformer windings, etc., must be low; and this is of still greater importance if it is desired to *over-compound* the rotaries. If the distance of transmission is great, or the voltage so low as to involve a considerable drop of pressure in the cables, then this method of regulation is unsuitable.

The chief objection to the compound winding of rotary converters as above described is that the power factor of the polyphase supply is continually changing, and this makes it difficult to maintain constant pressure at the receiving ends of the transmission lines. Mr. A. C. Eborall, in his paper on polyphase sub-station machinery, previously referred to, states that, in practice, the self-induction in the polyphase leads and the amount of the field excitation would be so adjusted as to make the armature current at no load a lagging one equal to about 30 or 40 per cent. of the full-load current. As the load comes on the machine, the power factor improves and becomes nearly unity at half full load, after which, with a further increase of load (and, therefore, of field ampere turns), the armature current would be in advance of the applied E.M.F.

(101) **"Hunting" of Rotary Converters**.—The

satisfactory running of synchronous motors, and, therefore, also of rotary converters, depends largely upon the engines in the generating station. If these have not sufficient flywheel effect, or are otherwise defective in the matter of imparting a uniform angular velocity to the generators, then trouble is to be looked for in connection with synchronous machines taking current from the supply. By uniformity of angular velocity is meant constant speed *per revolution*; and a high-speed engine will necessarily be more satisfactory in this respect than a low-speed engine: it also follows that "phase swinging" or "hunting" troubles are not likely to arise when the generators are driven by steam or water turbines.

Sometimes rotary converters will "hunt" even when the engines are in all respects suitable for the work they have to do. A slight oscillation may be started through various causes, and — especially if there are several machines working in parallel — this may tend to increase, owing to periodic distortion of the field produced by the "swinging" of the armature, and the resulting variations in the phase angle of the current fed into the slip rings. It is, therefore, advisable to design the rotaries to check, as far as possible, this tendency to "phase swinging" which may result in the machines falling out of synchronism, not to mention the probability of severe sparking at the commutator brushes.

Damping coils or "amortisseurs" suggest themselves at once as a suitable means of preventing distortion of the magnetic field under the pole faces; and these coils may take the form of extra heavy high-conductivity metal flanges supporting the field coils at the ends nearest to the pole faces. The use of such castings, or of heavy short-circuited copper conductors between the pole shoes, may lower the efficiency from 1 to 1½ per cent.; but this is of no consequence compared with the importance of steady running and freedom from hunting troubles. As a matter of fact, solid cast-iron pole faces act in a similar manner to the damping coils, and are equally, if not more, effective.

Hunting is far less likely to occur with low than with high periodicities, and 60 may be considered as the upper limit, beyond which it would be inadvisable to work rotary converters: sparking troubles may be expected, and — in any case for large units—motor-generators will generally be found preferable at this, or higher periodicities. The best frequency for rotary converters lies between 25 and 40. The machines would work satisfactorily a lower frequencies, but the cost of step-down transformers would be excessive.

Hunting troubles will be increased if the connections on the polyphase side are of high resistance, and, for this reason, rotary converters are generally found to work better when within a comparatively short distance of the generating plant than when placed at the distant end of fairly long transmission cables.

Although a rotary converter, when transforming polyphase into direct currents, will behave very much in the same manner as any synchronous polyphase motor, there is one important difference which should not be overlooked, as it may account for several *peculiarities* of this class of machinery. The difference referred to is in the amount of the armature reaction, which may be considerable in a synchronous motor, but is practically negligible in the rotary converter: this, indeed, explains why the latter is more liable to hunt than the synchronous motor.

In conclusion, it should be stated that, although the rotary converter has been considered throughout as taking the supply from the polyphase mains (which is the usual arrangement), the machine is reversible, and can be supplied with direct current at the brushes, in which case it will give out alternating currents at the slip rings. If such a machine be used for supplying alternating or polyphase currents in parallel with other machines, difficulties arise owing to the absence of armature reaction. Any lagging currents on the alternating-current side tend to weaken the field, and generally produce a condition of instability, requiring special methods of field excitation to remedy the trouble and prevent the rotary falling out of step.

NOTE I

COEFFICIENT OF SELF-INDUCTION.

The coefficient of self-induction, which is generally denoted by the symbol L, is a quantity, the use of which leads to some simplifications when it is permissible to assume a complete absence of iron in the magnetic circuit; for in this case L is constant, and may be defined as the amount of *self-enclosing* of magnetic lines by the circuit when the current has unit value. This coefficient of self-induction takes into account the number of times the total induction, **N**, is threaded through the circuit: for instance, if a circuit makes one or two turns upon itself, it will be the same thing—as regards the magnitude of the induced E.M.F.—as if it formed only one loop, but enclosed two or three times the amount of magnetism which we have called the self-induction. Thus, if we consider a coil of wire of S turns, the self-induction (if there is no iron) will be proportional to the ampere-turns S × C; but the coefficient of self-induction, L, will be proportional to S^2. If the coil of wire had consisted of only a single turn, the coefficient of self-induction might have been defined as the number of magnetic lines which would be threaded through the circuit if one (absolute) unit of current were flowing. Let us denote this by $\mathbf{N}_o$; then, if, instead of having only *one turn*, the coil be supposed to consist of S turns, the self-induction will be S times N_o, and the amount of *self-enclosing* of magnetic lines will be $\mathsf{N}_o\ S^2$. This may be written N S, where N stands for the actual number of lines threaded through a coil of S turns when unit current is passing through the coil.

When there is iron in the path of the magnetic lines, the coefficient of self-induction will no longer be constant, but will depend, to a certain extent, upon the value of the current. We can, however, still consider L as a coefficient which, when multiplied by the maximum value of the current, will give us the total amount of *self-enclosing* of magnetic lines, if we put

$$L = \frac{\mathsf{N}\ S}{C},$$

where C stands for the *maximum* value of the alternating current actually flowing in the circuit; and this definition of L applies equally well to the case of a circuit in which there is no iron, only, as N is then proportional to C, L will have a constant value which does not depend upon the current.

For reasons which it is not necessary to explain, the *practical* unit coefficient of self-induction is 10^9 times greater than the absolute C.G.S. unit, and the name *henry* applies to this practical unit coefficient of self-induction.

NOTE II

ON THE RELATION BETWEEN MAGNETIC FLUX AND INDUCED E.M.F. IN A CIRCUIT CONVEYING AN ALTERNATING CURRENT.

Let **N** be the total amount of magnetic flux through a coil of wire due to a certain definite maximum value of the alternating current. We need not concern ourselves here with the relation between the current and the total flux; but this can be approximately predetermined in the usual way (as adopted in dynamo and motor design), provided the material, arrangement, and measurements of the magnetic circuit are known.

Let S be the number of turns in the coil of wire, and n the periodicity of the current (number of cycles per second), then, since the total number of magnetic lines denoted by **N** are twice created and twice withdrawn during the course of one complete period, the *mean* value of the induced E.M.F. will be

$$E_m = 4\,\mathbf{N}\,S\,n,$$

and this equation is true whatever may be the shape of the current wave.

In the above formula E_m is the induced E.M.F. expressed in absolute C.G.S. units. If we wish to express the mean induced E.M.F., e_m, in *volts*, we must write

$$e_m = \frac{4\,\mathbf{N}\,S\,n}{100,000,000}$$

It is not, however, the *mean* value of the induced E.M.F. which we generally require to know, but its $\sqrt{\text{mean square}}$ value. Let us denote the ratio $\frac{\sqrt{\text{mean square volts}}}{\text{mean volts}}$ by the letter m, the value of which will depend upon the *shape* of the E.M.F. wave, and which, in fact, is the reciprocal of Dr. Fleming's *form factor*; we may then write

$$\textit{induced E.M.F.}\ \text{(in volts)} = \frac{m \times 4\ \mathbf{N}\ S\ n}{100{,}000{,}000}.$$

If the E.M.F. wave is a sine curve, the multiplier $m = 1{\cdot}11$, and as this is an assumption which leads to sufficiently accurate results when designing commercial choking coils, the formula may be written

$$\text{induced E.M.F. (in volts)} = \frac{4{\cdot}44\ \mathbf{N}\ S\ n}{100{,}000{,}000}\ . \quad (1)$$

In the event of there being no iron in the magnetic circuit, it is permissible to express the quantity $\mathbf{N} \times S$ in terms of the *coefficient of self-induction*, and the current in the coil.

In Note 1 it is explained how

$$L = \frac{\mathbf{N}\ S}{C},$$

which may be written

$$\mathbf{N}\ S = L \times C,$$

where L and C represent the coefficient of self-induction and the *maximum* value of the current, both in *absolute C.G.S. units.*

If L and C stand for the *practical* units, we must write

$$N\ S = L \times 10^9 \times \frac{C}{10}.$$

Again, if we assume the current wave to be a sine curve (which will be the case—in the absence of iron in the circuit—if the E.M.F. wave is a sine curve) it can be shown that the *maximum* value is $\sqrt{2}$ times greater than the $\sqrt{\text{mean square}}$ value. Let us put *c* for this latter value of the current in the above equation, and we get

$$N\ S = L\, c \times 10^8 \times \sqrt{2},$$

and substituting this value of N S in equation (1) we get

$$\text{induced E.M.F. (in volts)} = 6{\cdot}283\ n\ L\ c.$$

Here the multiplier 6·283 is none other than the better known $2\ \pi$, and we may write the equation

$$e = 2\ \pi\ n\ L\ c,$$

which is well known in connection with the analytical treatment of self-induction problems.

NOTE III.

Losses in Condensers.

Just as, in the case of self-induction, there is no such thing in practice as a truly *wattless* magnetising current, so also, in regard to condensers, the current will not be exactly 90 deg. in advance of the applied potential difference at the condenser terminals.

The losses in condensers are not easily determined with accuracy, and, moreover, they depend largely upon the condition of the dielectric, especially as regards moisture — they are found to be reduced after the condenser has been thoroughly dried—but in no case are they likely to be of a serious nature (their effect is to very slightly raise the temperature of the dielectric). A commercial condenser behaves almost exactly as if a small non-inductive resistance (which may be termed the *spurious* resistance of the condenser) were joined in series with a theoretically perfect condenser—*i.e.*, one in which there is no loss of energy when connected to a source of alternating E.M.F.

As regards the actual amount of such losses in practical working; from a number of tests made on concentric cables, both with vulcanised rubber and with paper insulation, it would appear that the *power factor* varies between ·02 and ·1—that is to say, the actual work done in heating up the insulation of the cable is from 2 to 10 per cent. of the *apparent watts*, this latter quantity being the product of the applied volts and the capacity current

The losses in cables, or condensers with solid dielectrics, are generally assumed to be proportional to the *square* of the applied potential difference, and this appears to be confirmed by results obtained from recent tests.

NOTE IV.

METHOD OF DRAWING THE COMPLETE VECTOR DIAGRAM FOR A POLYPHASE INDUCTION MOTOR FROM THREE SETS OF MEASUREMENTS MADE ON THE MACHINE.

The three sets of measurements required, in order that all conditions of working may be predetermined, are as follows:

(1) The current, volts, and true watts per phase of the stator winding when the motor is running light, at practically synchronous speed.

(2) The current, volts, and true power per phase of the stator winding, with rotor short-circuited and held in fixed position to prevent turning.

(3) The resistance per phase of the stator winding.

The set of readings (1) should be made with the full working voltage on primary terminals; but the set of readings (2) are preferably made with reduced volts at primary terminals, such as to allow of a current not exceeding the normal full-load current to pass: in both cases the frequency must be the same as that for which the motor is designed, and on which it will have to work.

The general diagram, Fig. 95, which appeared on p. 268, has been reproduced here, together with Fig. 95A, which explains the construction from the above data.

Draw $\overline{OA}$ (Fig. 95A) in any direction, to represent the magnetising current when motor is running light;

and $\overline{O\ B}$, to represent the applied potential difference at stator terminals; the angle between these two vectors being α, which must be such that cos α = the power factor, or ratio of true watts to apparent watts.

Now draw B H parallel to O A, and of such a length as to represent, to the same scale as O B, the volts lost in overcoming the ohmic resistance of the stator windings: it will be equal to (O A) × R, where R stands for the resistance per phase of the primary winding, referred to under test (3). Join H O and produce to K, making O K equal to O H. This gives us $\overline{O\ K}$ for the E.M.F. of self-induction (which, in the rotor, is balanced by the E.M.F. of

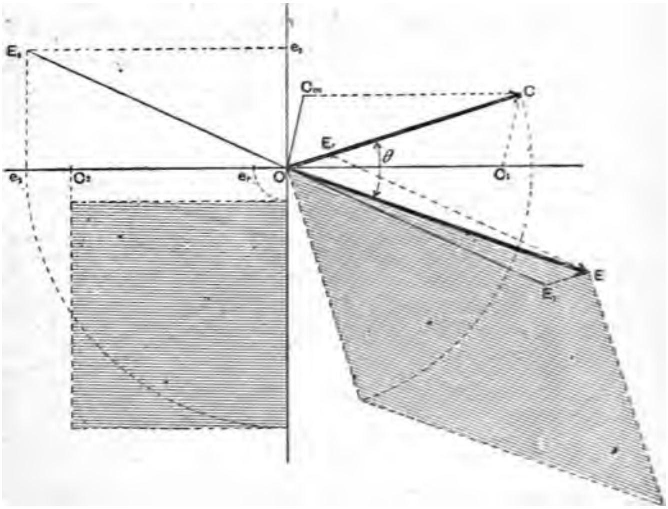

FIG. 95.

rotation in the magnetic field; with the result that there is no appreciable current in the rotor conductors). It is this pressure vector which — in Fig. 95—is supposed to be of constant value. We can, therefore, draw the dotted circles, of radius O K or O H, from the centre, O, and—under all conditions of load—the vector representing the total induced E.M.F. per phase of the primary circuit must lie on the arc of circle described through K, while its balancing E.M.F. component of the total primary pressure will lie on the arc of circle described through H.

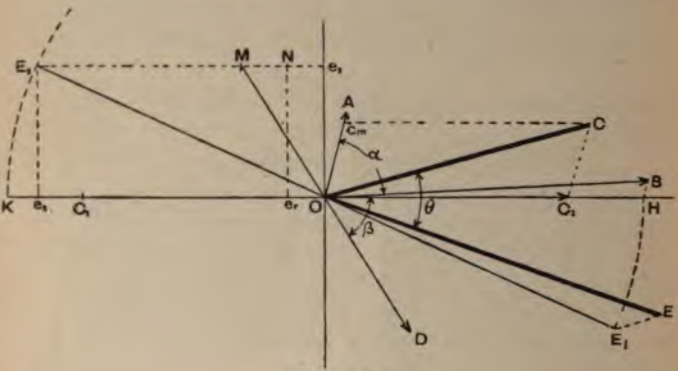

FIG. 95A.

We will now turn our attention to the second set of measurements—*i.e.*, the terminal volts and the true watts corresponding to a definite primary current C_1, obtained when the rotor is short-circuited and clamped in position to prevent rotation.

Note, in the first place, that the magnetising *current* (in the phase O A) will be so small as to be negligible, being merely such as will induce the very low volts, e_r, in rotor, required to overcome the resistance of the windings. It follows that the total primary current, C_1, is almost exactly equal and opposite to the rotor current, C_2, and these current vectors must, therefore, be drawn respectively in the phases O H and O K.

Now draw $\overline{O\ D}$ in advance of $\overline{O\ C_1}$, to represent the measured potential difference per phase at primary terminals, the angle β between the two vectors being such that cos β = the ratio of (measured) true watts to apparent watts, and produce D O to M, making O M = O D.

The vector $\overline{O\ D}$ may be considered as being made up of two components at right angles; one of these, in phase with the current, and equal to M e_3, is required to overcome the resistance of stator and rotor windings, while the other (equal but opposite to $\overline{O\ e_3}$) is required to balance the E.M.F. of self-induction due to the leakage magnetism which does not enter the rotor. In this manner we obtain the length of the vector $\overline{O\ e_3}$ corresponding to a given secondary current, C_2; and for any other value of the rotor current, this leakage E.M.F. vector may be assumed to vary directly as the current.

Referring again to the vector $\overline{e_3\ M}$, this is made up of two parts, M N and e_3 N, representing the E.M.F. components required to overcome primary and secondary resistances respectively. It is easy to calculate M N, since it is equal to (O C_1) × R, where R = primary resistance per phase, and this leaves e_3 N, or $\overline{O\ e_r}$ for the E.M.F. required to overcome the secondary, or rotor resistance. (It is interesting to note that we have, in this way, arrived at the equivalent resistance of the rotor conductors without making a direct measurement of same, which, indeed, would be impossible in the case of a squirrel-cage rotor).

In order to complete the diagram corresponding to the working conditions when the rotor current is C_2, produce e_3 M (parallel to O K) until it meets the dotted circle at E_2: drop the perpendicular $E_2 e_2$ on to O K, and produce E_2 O to E_1, making O E_1 equal to O E_2.

The vector $\overline{O\ e_2}$ is the induced E.M.F. in rotor under working conditions, and it will require a magnetising current $\overline{O\ C_m}$ to produce it, of such a value that O C_m bears the same relation to O A as O e_2 bears to O K. This enables us to obtain the primary current vector $\overline{O\ C}$. Now draw E_1 E parallel to O C, and equal to (O C) × R, where R = primary resistance. Join O E: then $\overline{O\ C}$ and $\overline{O\ E}$ are the vectors representing the primary current and potential difference at terminals which will be required, under working conditions, when the rotor current has the value C_2; the power factor being evidently cos θ, where θ is the angle subtended by these vectors.

In this manner the complete diagram similar to Fig. 95 can be constructed from the three sets of measurements referred to above; and it is evidently only necessary to carry out the construction for one more value of the rotor current (equal to about 2 C_2)—no additional tests being required for this purpose — in order to obtain three points on the circle diagram, which can then be constructed in *all respects* as shown in Fig. 99 on p. 281

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